PQ - Sum Divisor Cordial graphs

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Abstract:
Graph labeling is an assignment of integers to the vertices or edges or both depending on certain conditions. A graph G with p vertices and q edges is said to admit PQ- sum divisor cordial labeling if the labeling h from V(G) to {1, 2, ..., p} induces a mapping 

\[ h^*: E(G) \rightarrow \{0, 1\} \]

as 

\[ h^*(xy) = \begin{cases} 
1 & \text{if } 2 \left( P_{xy} + Q_{xy} \right) \\
0 & \text{otherwise} 
\end{cases} \]

with the condition that \( |e_h^*(0) - e_h^*(1)| \leq 1 \), where \( e_h^*(k) \) is the number of edges labeled with k. A graph which admits a PQ- sum divisor cordial labeling is called a PQ- sum divisor cordial graph. In this paper, we prove that the path \( P_n \), cycle \( C_n \), star graph \( K_{1,n} \), bistar graph \( B_{m,n} \), the subdivision graph of the star and bistar graphs \( S(K_{1,n}) \) and \( S(B_{m,n}) \), the splitting graph of the star graph \( S'(K_{1,n}) \), the fan graph \( F_{1,n} \), the vertex switching of the path and the cycle, the graphs \( P_n^2 \) and \( P_n \odot K_1 \) are PQ- sum divisor cordial graphs.

Keywords: labeling, cordial labeling, sum divisor cordial labeling.

1. INTRODUCTION

Labeling of a graph is an immense and vast area of research in the field of graph theory. If the vertices or edges or both of a graph are assigned values subject to certain conditions, then it is known as graph labeling. Cahit proposed the notion of cordial labeling in 1987 as a weaker version of graceful and harmonious labeling [1]. Let f be a function from the vertex set of G to \{0, 1\} and for each edge xy assign the label \(|f(x) - f(y)|\). The function f is called a cordial labeling of G if the number of vertices labeled with 0 and the number of vertices labeled with 1 differ at most by 1 and number of edges labeled with 0 and the number of edges labeled with 1 differ at most by 1. The notion of sum divisor cordial labeling was introduced by A. Lourdusamy and F. Patrick[4]. Let f be a bijection from the vertex set of G to \{1, 2, ..., |V(G)|\} and for each edge xy assign the
label 1 if 2 divides \( f(x) + f(y) \) and the label 0 otherwise. The function \( f \) is called a sum divisor cordial labeling of \( G \) if the number of edges labeled with 0 and the number of edges labeled with 1 differ at most by 1. Motivated by this we introduced PQ- sum divisor cordial labeling of graphs. In this section we provide a summary of definitions and notations required for our investigation.

**Definition 1.1.** The subdivision graph \( S(G) \) is obtained from a graph \( G \) by subdividing each edge of \( G \) with a vertex.

**Definition 1.2.** For a graph \( G \) the splitting graph \( S'(G) \) is obtained by adding a new vertex \( x' \) corresponding to every vertex \( x \) of \( G \) such that \( N(x) = N(x') \), where \( N(x) \) is the set of all vertices adjacent to \( x \) in \( G \).

**Definition 1.3.** The corona product \( G_1 \odot G_2 \) of two graphs \( G_1(p_1, q_1) \) and \( G_2(p_2, q_2) \) is defined as the graph obtained by taking one copy of \( G_1 \) and \( p_1 \) copies of \( G_2 \) and joining \( i \)th vertex of \( G_1 \) with an edge to every vertex in the \( i \)th copy of \( G_2 \).

**Definition 1.4.** The fan graph \( F_{m,n} \) is defined as the join \( \overline{K}_m + P_n \), where \( \overline{K}_m \) is the trivial graph on \( m \) vertices and \( P_n \) is the path graph on \( n \) vertices.

**Definition 1.5.** The square graph \( G^2 \) of a graph \( G \) is obtained from \( G \) by adding new edges between every two vertices having distance two in \( G \).

**Definition 1.6.** The vertex switching \( G_v \) of a graph \( G \) is the graph obtained by removing all the edges incident with the vertex \( v \) of \( G \) and joining the vertex \( v \) to every vertex which is not adjacent to \( v \) by an edge.

**Notation 1.7.** Let \( h \) be a vertex labeling and \( xy \in E(G) \). We denote \( P_{xy} = h(x)h(y) \) and \( Q_{xy} = \begin{cases} h(x) & \text{if } h(x) > h(y) \\ h(y) & \text{if } h(y) > h(x) \end{cases} \) for all \( xy \in E(G) \).

**Notation 1.8.** Let us denote \( e_h^*(k) \) = the number of edges labeled with \( k \).

**Definition 1.9.** Let \( G \) be a simple graph and \( h: V(G) \to \{1, 2, \ldots, |V(G)|\} \) be a bijection. For each edge \( xy \) assign \( h^*(xy) = \begin{cases} 1 & \text{if } 2 \mid (P_{xy} + Q_{xy}) \\ 0 & \text{otherwise} \end{cases} \). The labeling \( h \) is called a PQ- sum divisor cordial labeling if \( |e_h^*(0) - e_h^*(1)| \leq 1 \). A graph which admits a PQ- sum divisor cordial labeling is called a PQ- sum divisor cordial graph.
2. MAIN RESULTS

Theorem 2.1. The path graph $P_n$ is a PQ- sum divisor cordial graph.

Proof. Let $G$ be the path graph $P_n$. Let $x_i (1 \leq i \leq n)$ be the vertices of $G$.

Define $h : V(G) \to \{1, 2, \ldots, n\}$ by $h(x_1) = 1$, $h(x_2) = 2$ and

$$h(x_i) = \begin{cases} 
2i - 3 & \text{if } 3 \leq i \leq \left\lceil \frac{n}{2} \right\rceil + 1 \\
2(n - i) + 4 & \text{if } \left\lceil \frac{n}{2} \right\rceil + 2 \leq i \leq n 
\end{cases}$$

Then the labeling $h$ will induce the map $h^* : E(G) \to \{0, 1\}$. Here, the number of edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = \frac{n - 1}{2}$ if $n$ is odd and $e_{h^*}(0) = \left\lfloor \frac{n - 1}{2} \right\rfloor$, $e_{h^*}(1) = \left\lceil \frac{n - 1}{2} \right\rceil$ if $n$ is even. Thus $|e_{h^*}(0) - e_{h^*}(1)| \leq 1$ and hence $P_n$ is a PQ- sum divisor cordial graph.

Theorem 2.2. The complete graph $K_n$ is not a PQ- sum divisor cordial graph for all $n$.

Proof. Let $G$ be the complete graph $K_n$. Let $x_i (1 \leq i \leq n)$ be the vertices of $G$. Label the vertices of $G$ in any order.

Let $A = \{1, 2, 3, 7\}$. Then we have $e_{h^*}(1) > \left\lceil \frac{n(n - 1)}{4} \right\rceil$ for all $n \leq 17$ and $n \notin A$. Also, $e_{h^*}(1) < \left\lceil \frac{n(n - 1)}{4} \right\rceil$ for all $n \geq 18$. Thus, $|e_{h^*}(0) - e_{h^*}(1)| > 1$ for all $n \notin A$. Hence, $K_n$ is not a PQ-sum divisor cordial graph for all $n$.

Theorem 2.3. The cycle graph $C_n$ is a PQ- sum divisor cordial graph.

Proof. Let $G$ be the cycle graph $C_n$. Let $x_i (1 \leq i \leq n)$ be the vertices of $G$. Define $h : V(G) \to \{1, 2, \ldots, n\}$ as follows:

Case(i): $n$ is odd

Label the vertex $x_1$ by 1, $x_{\left\lfloor \frac{n}{2} \right\rfloor}$ by $n$, $x_{\left\lceil \frac{n}{2} \right\rceil}$ by $n - 1$ and the remaining vertices by

$$h(x_i) = \begin{cases} 
2i - 2 & \text{if } 2 \leq i \leq \left\lceil \frac{n}{2} \right\rceil \\
2(n - i) + 3 & \text{if } \left\lceil \frac{n}{2} \right\rceil + 2 \leq i \leq n 
\end{cases}$$

Then the labeling $h$ will induce the map $h^* : E(G) \to \{0, 1\}$. Also, we get $e_{h^*}(1) = \left\lfloor \frac{n}{2} \right\rfloor$ and $e_{h^*}(0) = \left\lceil \frac{n}{2} \right\rceil$.
Case(ii): n is even

In this case, label the vertex $x_i$ by 1, $x_{\frac{n}{2}+1}$ by $n - 1$, $x_{\frac{n}{2}+2}$ by n and the remaining vertices by $h(x_i) = \begin{cases} 2i - 2 & \text{if } 2 \leq i \leq \frac{n}{2} \\ 2(n - i) + 3 & \text{if } \frac{n}{2} + 3 \leq i \leq n \end{cases}$.

Then the number of edges labeled with 0 and 1 are $e_{h^*}(0) = e_{h^*}(1) = \frac{n}{2}$.

Thus in each case, we have $|e_{h^*}(0) - e_{h^*}(1)| \leq 1$. Hence, $C_n$ is a PQ- sum divisor cordial graph.

**Theorem 2.4.** The graph $P_n^2$ is a PQ- sum divisor cordial graph.

**Proof.** Let $G = P_n^2$. Let $x_i (1 \leq i \leq n)$ be the vertices of G. Then

$E(G) = \{x_i, x_{i+1} : 1 \leq i \leq n - 1\} \cup \{x_i, x_{i+2} : 1 \leq i \leq n - 2\}$. Define $h : V(G) \rightarrow \{1, 2, \ldots, n\}$ as follows:

Case(i): n is odd

$h(x_i) = \begin{cases} 2i - 1 & \text{if } 1 \leq i \leq \frac{n}{2} \\ 2(n - i) + 2 & \text{if } \frac{n}{2} + 2 \leq i \leq n \end{cases}$, $h\left(x_{\frac{n}{2}}\right) = n - 1$ and $h\left(x_{\frac{n}{2}+1}\right) = n$.

Here, the labeling h will induce the map $h^* : E(G) \rightarrow \{0, 1\}$. Also, we get $e_{h^*} (1) = n - 1$ and $e_{h^*} (0) = n - 2$.

Case(ii): n is even

$h(x_i) = \begin{cases} 2i - 1 & \text{if } 1 \leq i \leq \frac{n}{2} \\ 2(n - i) + 2 & \text{if } \frac{n}{2} + 1 \leq i \leq n \end{cases}$

In this case, we have $e_{h^*} (1) = n - 1$ and $e_{h^*} (0) = n - 2$.

Thus in each case, we have $|e_{h^*} (0) - e_{h^*} (1)| \leq 1$. Hence, $P_n^2$ is a PQ- sum divisor cordial graph.

**Theorem 2.5.** The star graph $K_{1,n}$ is a PQ- sum divisor cordial graph.

**Proof.** Let G be the star graph $K_{1,n}$. Let $x, x_i (1 \leq i \leq n)$ be the vertices of G. Define $h : V(G) \rightarrow \{1, 2, \ldots, n + 1\}$ as follows:

Case(i): $n \equiv 0(\text{mod } 4)$
Label the vertex $x$ by $4$ and $x_i (1 \leq i \leq n)$ by $1, 2, 3, 5, 6, \ldots, n + 1$ in any order. Then the number of edges labeled with $0$ and $1$ are $e_{k^*}(0) = e_{k^*}(1) = \frac{n}{2}$. 

Case(ii): $n \equiv 1, 2, 3 (\text{mod} \ 4)$

Label the vertex $x$ by $2$ and $x_i (1 \leq i \leq n)$ by $1, 3, 4, \ldots, n + 1$ in any order. Then the number of edges labeled with $0$ and $1$ are $e_{k^*}(0) = \left\lceil \frac{n}{2} \right\rceil$ and $e_{k^*}(1) = \left\lfloor \frac{n}{2} \right\rfloor$.

In both cases, we have $|e_{k^*}(0) - e_{k^*}(1)| \leq 1$. Hence, $K_{1,n}$ is a PQ- sum divisor cordial graph.

**Theorem 2.6.** The fan graph $F_{1,n}$ is a PQ- sum divisor cordial graph.

**Proof.** Let $G$ be the fan graph $F_{1,n}$. Let $x, x_i (1 \leq i \leq n)$ be the vertices of $G$. Then $E(G) = \{xx_i : 1 \leq i \leq n\} \cup \{x_ix_{i+1} : 1 \leq i \leq n - 1\}$. Define $h : V(G) \rightarrow \{1, 2, \ldots, n + 1\}$ by $h(x) = 1$ and $h(x_i) = i + 1$. Then the labeling $h$ will induce the map $h^* : E(G) \rightarrow \{0, 1\}$ and we get $e_{k^*}(1) = n$, $e_{k^*}(0) = n - 1$.

Here, $|e_{k^*}(0) - e_{k^*}(1)| \leq 1$. Hence, $F_{1,n}$ is a PQ- sum divisor cordial graph.

**Theorem 2.7.** The comb graph $P_n \odot K_1$ is a PQ- sum divisor cordial graph.

**Proof.** Let $G$ be the comb graph $P_n \odot K_1$. Let $x, y_i (1 \leq i \leq n)$ be the vertices of $G$. Then $E(G) = \{x_iy_i : 1 \leq i \leq n\} \cup \{x_ix_{i+1} : 1 \leq i \leq n - 1\}$.

Define $h : V(G) \rightarrow \{1, 2, \ldots, 2n\}$ by $h(x_i) = 2i - 1$ and $h(y_i) = 2i$. Then the induced map $h^* : E(G) \rightarrow \{0, 1\}$ satisfies $e_{k^*}(1) = n$ and $e_{k^*}(0) = n - 1$. Here, $|e_{k^*}(0) - e_{k^*}(1)| \leq 1$.

Hence, $P_n \odot K_1$ is a PQ- sum divisor cordial graph.

**Theorem 2.8.** The bistar graph $B_{m,n}$ is a PQ- sum divisor cordial graph.

**Proof.** Let $G$ be the bistargraph $B_{m,n}$. Let $x, y, x_i (1 \leq i \leq m)$, $y_j (1 \leq j \leq n)$ be the vertices of $G$. Without loss in generality we may assume that $m \geq n$. Define $h : V(G) \rightarrow \{1, 2, \ldots, m + n + 2\}$ as follows:

Case(i): Both $m$ and $n$ are odd

$$h(x_i) = \begin{cases} 
4 \left\lfloor \frac{i}{2} \right\rfloor - 1 & \text{if } i \text{ is odd} \\
2i & \text{if } i \text{ is even}
\end{cases} \quad \text{for } 3 \leq i \leq n + 1 \quad \text{and} \quad h(y_j) = \begin{cases} 
4 \left\lfloor \frac{j}{2} \right\rfloor + 1 & \text{if } j \text{ is odd} \\
2j + 2 & \text{if } j \text{ is even}
\end{cases} \quad \text{for } 1 \leq j \leq n
$$

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Then the labeling \( h \) will induce the map \( h^*: E(G) \to \{0, 1\} \). Also the number of edges labeled with 0 and 1 are 
\[
e_{h^*}(0) = \left\lfloor \frac{m+n+1}{2} \right\rfloor \quad \text{and} \quad e_{h^*}(1) = \left\lceil \frac{m+n+1}{2} \right\rceil.
\]
Case(ii): Both \( m \) and \( n \) are even
In this case, label the vertices of \( G \) as in case(i). Then the number of edges labeled with 0 and 1 are
\[
e_{h^*}(0) = \left\lfloor \frac{m+n+1}{2} \right\rfloor \quad \text{if} \quad n \equiv 2(\text{mod} \ 4) \quad \text{and} \quad e_{h^*}(1) = \left\lceil \frac{m+n+1}{2} \right\rceil \quad \text{if} \quad n \equiv 0(\text{mod} \ 4)
\]
Case(iii): \( m \) is odd and \( n \) is even
If \( m \equiv 1(\text{mod} \ 4) \), label the vertices of \( G \) as in case(i).
If \( m \equiv 3(\text{mod} \ 4) \) and \( n \equiv 2(\text{mod} \ 4) \), label the vertices of \( G \) by
\[
h(x_i) = \begin{cases} 2i & \text{if } i \text{ is odd} \\ 2i+1 & \text{if } i \text{ is even} \end{cases} \quad \text{for } 2 \leq i \leq n
\]
\[
h(y_j) = \begin{cases} 2j+2 & \text{if } j \text{ is odd} \\ 2j+3 & \text{if } j \text{ is even} \end{cases} \quad \text{for } 2 \leq j \leq n
\]
If \( m \equiv 3(\text{mod} \ 4) \) and \( n \equiv 0(\text{mod} \ 4) \), label the vertices of \( G \) by
\[
h(x_i) = \begin{cases} 2i+2 & \text{if } i \text{ is odd} \\ 2i+3 & \text{if } i \text{ is even} \end{cases} \quad \text{for } 2 \leq i \leq n-1
\]
\[
h(y_j) = \begin{cases} 2j & \text{if } j \text{ is odd} \\ 2j+1 & \text{if } j \text{ is even} \end{cases} \quad \text{for } 2 \leq j \leq n
\]
Here, we observe that \( e_{h^*}(0) = e_{h^*}(1) = \frac{m+n+1}{2} \).
Case(iv): \( m \) is even and \( n \) is odd
If \( (m \equiv 0(\text{mod} \ 4) \text{ and } n \equiv 3(\text{mod} \ 4)) \text{ or } (m \equiv 2(\text{mod} \ 4) \text{ and } n \equiv 1(\text{mod} \ 4)) \), label the vertices of \( G \) as in case(i). If \( (m \equiv 0(\text{mod} \ 4) \text{ and } n \equiv 1(\text{mod} \ 4)) \text{ or } (m \equiv 2(\text{mod} \ 4) \text{ and } n \equiv 3(\text{mod} \ 4)) \), label the vertices of \( G \) as in the case \( m \equiv 3(\text{mod} \ 4) \) and \( n \equiv 2(\text{mod} \ 4) \) of case(iii).
Then the number of vertices and edges labeled with 0 and 1 are \( e_{h^*}(0) = e_{h^*}(1) = \frac{m+n+1}{2} \).
In each cases, \( |e_{h^*}(0) - e_{h^*}(1)| \leq 1 \). Hence, \( B_{m,n} \) is a PQ- sum divisor cordial graph.

**Theorem 2.9.** The graph \( S(K_{1,n}) \) is a PQ- sum divisor cordial graph.

**Proof.** Let \( G \) be the subdivision graph of the star graph \( K_{1,n} \). Let \( x_i, (1 \leq i \leq n) \) be the vertices of \( K_{1,n} \) and let \( x_i^+, (1 \leq i \leq n) \) be the vertices which subdivides the edges \( xx_i, (1 \leq i \leq n) \). Define \( h: V(G) \to \{1, 2, \ldots, 2n+1\} \) by \( h(x) = 1, h(x_i) = 2i+1 \text{ and } h(x_i^+) = 2i \). Then \( h \) will induce the map \( h^*: E(G) \to \{0,1\} \) and we get \( e_{h^*}(0) = e_{h^*}(1) = n \).
Here, \( |e_h(0) - e_h(1)| \leq 1 \). Hence, \( S(K_{1, n}) \) is a PQ- sum divisor cordial graph.

**Theorem 2.10.** The graph \( S(B_{m, n}) \) is a PQ- sum divisor cordial graph.

Proof. Let \( G \) be the subdivision graph of the bistar \( B_{m, n} \). Let \( x, y, x_i (1 \leq i \leq m) \) and \( y_j (1 \leq j \leq n) \) be the vertices of \( B_{m, n} \). Let \( z \) be the vertex which subdivides the edge \( xy \) and let \( x'_j(1 \leq i \leq m), y'_j (1 \leq j \leq n) \) be the vertices which subdivides the edges \( xx_i (1 \leq i \leq m) \) and \( yy_j (1 \leq j \leq n) \) respectively. Define \( h : V(G) \to \{1, 2, \ldots, 2(m + n) + 3\} \) as follows:

Case(i): \( n \) is even

\[
\begin{align*}
h(x) &= 1, \quad h(y) = 2, \quad h(z) = 3, \quad h(x_i) = 2i + 3, \quad h(x'_i) = 2(i + 1) \quad \text{for} \ 1 \leq i \leq m, \\
h(y_j) &= \begin{cases} 2(m + j) + 2 & \text{if} \ j \text{ is odd} \\ 2(m + j) + 3 & \text{if} \ j \text{ is even} \end{cases}
\end{align*}
\]

**Case(ii):** \( n \) is odd

\[
\begin{align*}
h(x) &= 1, \quad h(y) = 2, \quad h(z) = 3, \quad h(x_i) = 2i + 3, \quad h(x'_i) = 2(i + 1) \quad \text{for} \ 1 \leq i \leq m, \\
h(y_j) &= \begin{cases} 2(m + j) + 4 & \text{if} \ j \text{ is odd} \\ 2(m + j) + 1 & \text{if} \ j \text{ is even} \end{cases}
\end{align*}
\]

In this case, label the vertices \( x, y, z, x_i, x'_i (1 \leq i \leq m - 2), y_j, y'_j (1 \leq j \leq n - 2) \) as in case(i). Also label \( h(y_{n-1}) = 2(m + n) + 3, \quad h(y_n) = 2(m + n + 1), \quad h(y_{n-1}') = 2(m + n) - 1 \) and \( h(y_n') = 2(m + n) + 1 \).

In each case, the number of edges labeled with 0 and 1 are \( e_h(0) = e_h(1) = m + n + 1 \). Hence, \( S(B_{m, n}) \) is a PQ- sum divisor cordial graph.

**Theorem 2.11.** The graph \( S'(K_{1,n}) \) is a PQ- sum divisor cordial graph.

Proof. Let \( G \) be the splitting graph of the star graph \( K_{1,n} \). Let \( x, x_i (1 \leq i \leq n) \) be the vertices of \( K_{1,n} \) and let \( x', x'_i (1 \leq i \leq n) \) be the added vertices corresponding to \( x, x_i (1 \leq i \leq n) \) to form \( G \). Define \( h : V(G) \to \{1, 2, \ldots, n\} \) as follows:

Case(i): \( n \equiv 0(\text{mod} \ 4) \)

\[
\begin{align*}
h(x) &= 1, \quad h(x_1) = 3, \quad h(x) = 2, \quad h(x') = 4, \\
h(x_i) &= \begin{cases} 4 \left(\frac{i}{2}\right) - 1 & \text{if} \ i \text{ is odd} \\ 2i & \text{if} \ i \text{ is even} \end{cases} \quad \text{for} \ 3 \leq i \leq n \quad \text{and} \quad h(x'_i) &= \begin{cases} 4 \left(\frac{i}{2}\right) + 1 & \text{if} \ i \text{ is odd} \\ 2i + 2 & \text{if} \ i \text{ is even} \end{cases} \quad \text{for} \ 1 \leq i \leq n
\end{align*}
\]

Here, \( |e_h(0) - e_h(1)| \leq 1 \). Hence, \( S(K_{1,n}) \) is a PQ- sum divisor cordial graph.
Here, the labeling $h$ will induce the map $h^*: E(G) \to \{0, 1\}$. Also the number of edges labeled with 0 and 1 are $e_{h'}(0) = e_{h'}(1) = \frac{3n}{2}$.

Case (ii): $n \equiv 1, 2, 3, (\text{mod} \ 4)$

$h(x_i) = 1$, $h(x'_i) = 3$, $h(x) = 2$, $h(x') = 4,$

$$h(x_i) = \begin{cases} 2i + 2 & \text{if } i \text{ is odd} \\ 2i + 3 & \text{if } i \text{ is even} \end{cases} \text{ for } 2 \leq i \leq n \quad \text{and} \quad h(x'_i) = \begin{cases} 2i & \text{if } i \text{ is odd} \\ 2i + 1 & \text{if } i \text{ is even} \end{cases} \text{ for } 1 \leq i \leq n$$

Then the number of edges labeled with 0 and 1 are $e_{h'}(0) = e_{h'}(1) = \frac{3n}{2}$ if $n \equiv 2 (\text{mod} \ 4)$.

Also $e_{h'}(0) = \left\lfloor \frac{3n}{2} \right\rfloor$ and $e_{h'}(1) = \left\lceil \frac{3n}{2} \right\rceil$ if $n$ is odd.

Here, $|e_{h'}(0) - e_{h'}(1)| \leq 1$. Hence, $S'(K_{1,n})$ is a PQ-sum divisor cordial graph.

**Theorem 2.12.** Vertex switching of a cycle $C_n$ admits PQ-sum divisor cordial labeling.

Proof. Let $G$ be the cycle graph $C_n$ and let $G_{x_i}$ be the graph obtained from $G$ by switching a vertex $x$ of $G$. Let $x_i (1 \leq i \leq n)$ be the vertices of $G$. Without loss in generality we may assume that $x = v_1$. Define $h: V(G_{x_i}) \to \{1, 2, \ldots, n\}$ by $h(x_i) = i$. Then the labeling $h$ will induce the map $h^*: E(G) \to \{0, 1\}$. Also the number of edges labeled with 0 and 1 are $e_{h'}(0) = n - 2$ and $e_{h'}(1) = n - 3$.

Here, $|e_{h'}(0) - e_{h'}(1)| \leq 1$. Hence, vertex switching of a cycle $C_n$ is a PQ-sum divisor cordial graph.

**Theorem 2.13.** Vertex switching of a path graph $P_n$ admits PQ-sum divisor cordial labeling.

Proof. Let $G$ be a path graph $P_n$ and let $x_i (1 \leq i \leq n)$ be the vertices of $G$. Let $G_{x_i}$ be the graph obtained from $G$ by switching a vertex $x_i$ of $G$. Then we have $V(G) = V(G_{x_i})$.

Define $h: V(G_{x_i}) \to \{1, 2, \ldots, n\}$ by $h(x_i) = \begin{cases} k + 1 & \text{if } k < i \\ 1 & \text{if } k = i \\ k & \text{if } k > i \end{cases}$

Then the labeling $h$ will induce the map $h^*: E(G) \to \{0, 1\}$. Also the number of edges labeled with 0 and 1 are $e_{h'}(0) = e_{h'}(1) = n - 3$.

Here, $|e_{h'}(0) - e_{h'}(1)| \leq 1$. Hence, vertex switching of a path $P_n$ is a PQ-sum divisor cordial graph.
References