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Mathematical Modelling on Population Prediction

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ABSTRACT

We cannot imagine a good planet without population control. As the human population is increasing, so are the shortages of resources, such as lack of water, lack of land, and lack of trees. Due to this continuous increase in population, our natural resources are continuously depleting. Malthusian mathematical model and Logistic mathematical model were studied for how to population is growing. After extracting both these mathematical model solutions, the predicted population was extracted through Scilab software, and a graphical representation of the predicted population was done with the actual population and found that both the actual and predicted populations are similar in nature. Here the population assessment was done from 1961 to 2031. In 2010 the actual population was 1210.2 million while the population obtained from the exponential model was 1330.59 and the population obtained from the logistic model was 1024.86. There is a slight difference between the actual and predicted population, From these models government can predict the future population and arrange necessary resources such as economic as well as social.

Keywords: Population Growth, Exponential Model, Logistic Model, Carrying Capacity

INTRODUCTION

The population of a country is an essential element. The rate of increase and decrease in the population of a country determines the rate of development of the country. The government of the country would do socioeconomic and demographic development according to the rate of increase and decrease of population. On the basis of the results obtained from mathematical modelling, the government of the country can do various types of socio-economic and demographic development. The government of each country mobilizes various types of natural and other resources for the future according to the population growth (Wali A., 2011). Statisticians and mathematicians make a future projection from the previous data and draw the conclusion from the analysis ([Biswas MHA et al., 2011, Biswas HA, 2014 Islam MR, 2011). There are huge concerns about the large human population growth for the environmental, social, and economic development, which enhance all the problems in the population growth. Modelling is a boarding technique to compute real-life problems for future prediction (Wali A et al., 2012). Thus, modelling is a process in which a model is developed by taking a real-life situation and converting it into a differential equation after the

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solution of the differential equation, the future predicate can be made.

The main concept to solve the differential equation is to learn about the physical and real-life behaviour underlying the physical process that differential equation is believed to be the mathematical model (Biswas HA., 2012). Many researchers evaluate population growth through observation, experimentation, and modelling. Mathematical models of population growth are used to predict population growth, decay, food preservation, maximization of production as well as minimization of production, and many other applications (Akçakaya HR et al., 2000) . (Ulecio-Montoya et al., 2021) presented a nonlinear mathematical model that showed the population growth of the mosquito through the life cycle of the mosquito. They implemented the numerical solution of the differential equation using Scilab software. A population model is a special type of mathematical model, which is applied to study the population dynamics (Cohen JE., 1995).

In this paper, an overview of the population growth and population decay is studied through a mathematical model, which has been applied to predict the population of India so that the necessary action is applied in the field of maximization or minimization of production, food preservation as well as many other applications. Two simple deterministic models namely Malthusian growth and decay and the logistic mathematical model have been analysed to study the behaviour of population growth in the long-term population growth namely 10 years. It is also analysed that the logistic mathematical model is realistic as compared to the exponential growth mathematical model.

Exponential Population Growth Model:
The exponential growth model is based on the constant rate. This model is also known as the Malthusian growth model, the model is named after Thomas Robert Malthus (Deshotel D., 2013). This model is single species, population model. Let x(t) be the population of the number of individuals present at any time t. in this model x(t) is differentiable with respect to time t. let the growth of the population with a constant rate be x $(t + \delta t)$ any time $t + \delta t.$ Let the number of births 'b' and the number of deaths 'd' in the next interval of the time are directly proportional to the population at time t. The mathematically above statement can be written as

$$x(\mathrm{t} + \delta\mathrm{t}) - x(\mathrm{t}) = bx(\mathrm{t})\delta\mathrm{t} - dx(\mathrm{t})\delta\mathrm{t} \qquad \text{(i)}$$

On dividing equation (i) by δt and taking limit $\delta t \to 0$, we get

$$\frac{dx(t)}{dt} = (b - d)\mathrm{x(t)} \qquad \text{(ii)}$$

On integrating equation (ii) with respect to t , we get

$$x(t) = C\, e^{at} \qquad \text{(iii)}$$

where a=b-d

Let x(0) is initial population at time t=0, then the equation (iii) can be written as

$$x(t) = x(0)\, e^{-at} \qquad \text{(iv)}$$

This model is known as Malthusian growth model or exponential growth model.

Exponential Population Decay Model:
Exponential decay is happened when number of births 'b' is less than number of the deaths 'd' in the next interval of the time and both of them are directly proportional to the population at time t. The mathematical expression can be written as

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$x(\mathrm{t} + \delta\mathrm{t}) - x(\mathrm{t}) = dx(\mathrm{t})\delta\mathrm{t} - bx(\mathrm{t})\delta\mathrm{t}$ (v)

On dividing equation (v) by δt and taking limit $\delta t \to 0$, we get

$\frac{dx(t)}{dt} = -(b - d)\mathrm{x(t)}$ (vi)

On solving equation (vi), we get

$x(t) = -Ce^{-at}$ (vii)

Where a=b-d

Let x(0) is initial population at time t=0, then the equation (vi) can be written as

$x(t) = -x(0)\, e^{-at}$ (viii)

Hence equation (viii) is required solution of equation (v) under above initial condition.

Logistic model:

(P.F. Verhulst, 1838) assumed that every stable population has a saturation level. Logistic model was firstly augmented by Verhulst in this model they augmented by a multiplicative factor, $1 - \frac{x(t)}{k}$, where K is denoted by saturation level. He presented that the growth in population does not depend only population size but it depends upon it carrying capacity (Haque MM et al., 2012). Logistic population model can be expressed as

$\frac{dx(t)}{dt} = ax(t)\left(1 - \frac{x(t)}{K}\right)$ (ix)

Let x(0) is initial population at time t=0, then equation (ix) becomes

$x(t) = \frac{Kx(0)}{(K - x(0))e^{-at} + x(0)}$ (x)

Logistic model can be expressed as in the following facts

(1) If limit of t approaches to infinity, then population reaches to its carrying capacity i.e.

$$\log_{t\to\infty} x(t) = K$$

(2) The relative growth rate $\frac{1}{x}\frac{dx}{dt}$declines linearly with increasing population size

(3) When growth rate is maximum then $x(t)_{inf} = \frac{K}{2}$

Result Analysis:

To estimation of the future population of India as well as the world, first of all, one needs to determine the growth rate of the population using the exponential growth model. Taking x(0) i.e. initial population from the actual data and final population after 10 years. On putting the value of x(0) and x(t) and time t =10 in the exponential model to get the value of growth rate. After applying simple calculations, we get a growth rate equal to 17.8%. The predicated population value is obtained from 1961 to 2031 in the interval of 10 years. From table 1, it is analysed that the predicted value of the exponential model is similar to the actual population. The logistic model is also studied in this paper, it is analysed that the result obtained from the logistic model is similar to the actual population. Graphical representation is also done to compare the population growth in actual population and predicated population.

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Table 1: Variation between actual and projected populations

Census Year	Total Population of India (In million)	Predicted Population from Exponential Model (In million)	Predicted Population from Logistic Model (In million)
1961	439.2	--	
1971	548.2	548.2	548.2
1981	683.3	684.25	667.24
1991	846.4	854.07	790.65
2001	1028.7	1066.02	911.89
2011	1210.2	1330.59	1024.86
2021		1660.81	1125.06
2031		2072.99	1210.10

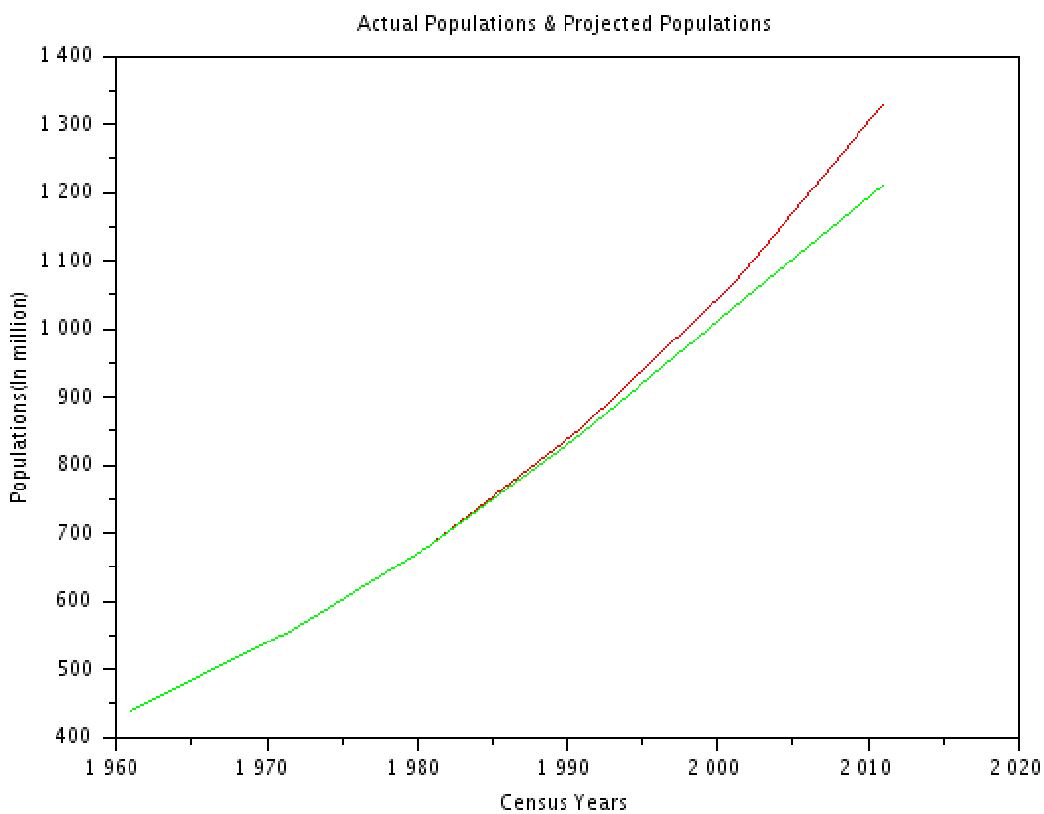

Figure 1: Variation Between Actual Populations and Exponential Projected Populations

From figure 1, it is concluded that the green colour (actual populations) population graph is monotonically increasing as time is increasing. In 1961 the population of India was 439.2 million but in 2011 it becomes 1210.2 million. In the graphical representation, the red colour shows the exponential projected populations. From figure 1, it is concluded that the population obtained from

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the exponential model is similar to the actual population. In 1961 the population obtained from the exponential model is the same as the actual population and from the graph, the population from 1961 to 2001 is similar in nature but from 2001 to 2021 the populations obtained from the projected model are slightly higher than the actual populations.

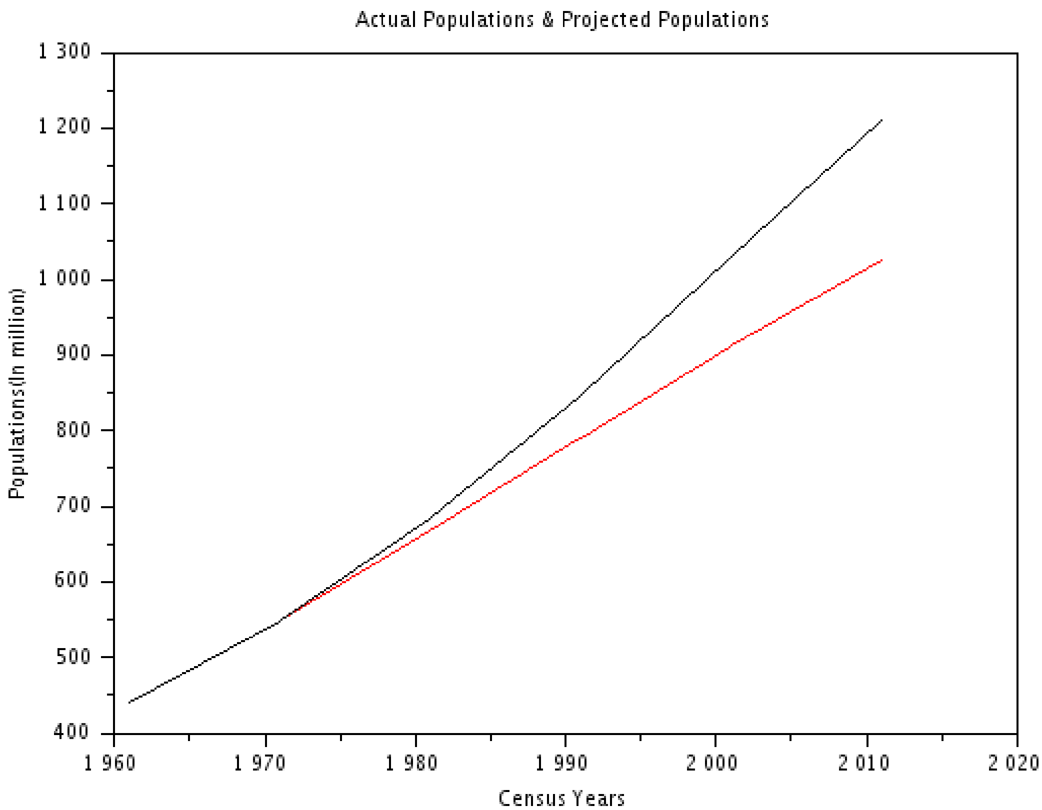

Figure 2: Variation Between Actual Populations and Logistic Projected Populations

From figure 2, it is concluded that the actual population and the predicated population obtained from the logistic model are similar in nature. In this model we took carrying capacity is equal to 1500 million, after 1500 million populations the curve will become s shaped since there is less competition in resources.

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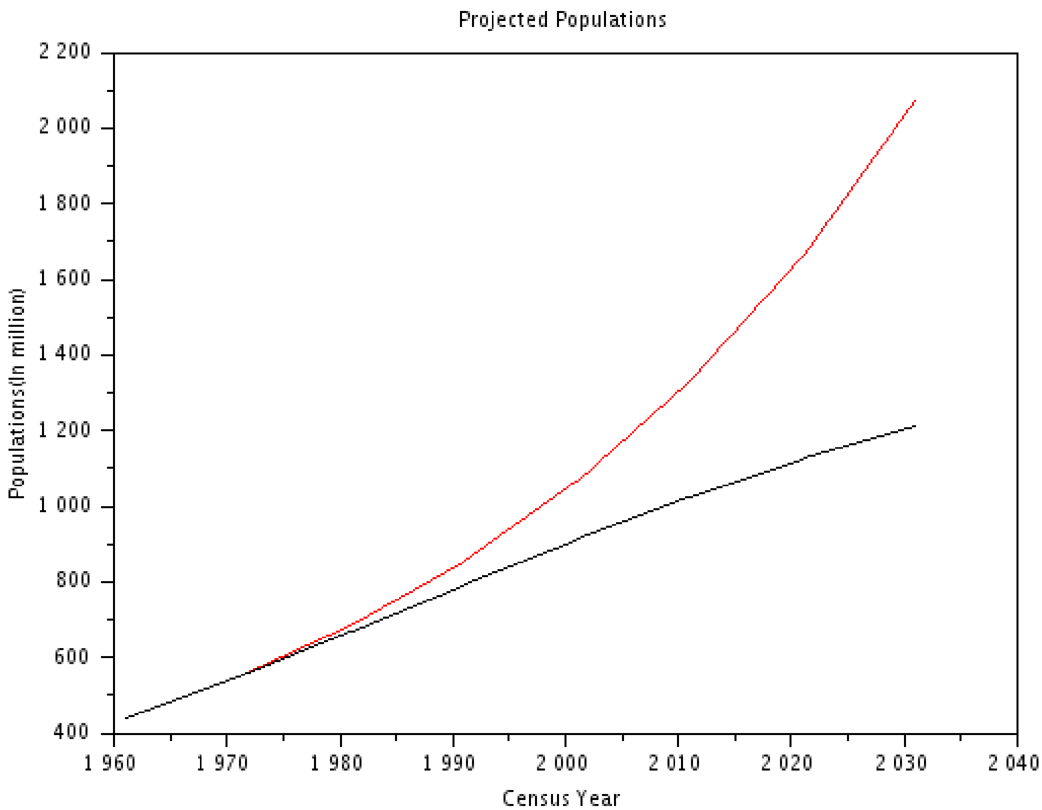

Figure 3: Variation between Exponential and logistic Model of population

In figure 3, the comparison is made between the exponential model and the logistic model, in the exponential model the growth curve (Red line) is very smooth and increasing steep over the time period. The exponential growth is deducted from various factors such as food limitations, diseases, and resources. As the environment has competition to get the resources then the population reaches the carrying capacity(K). Then this exponential growth is known as logistic growth.

Conclusion:

n this paper, the population obtained from both the models and compared with the actual population of India from 1961 to 2031, and it is analyzed that the population is growing at a very fast rate. To investigate the population behaviour the carrying capacity was considered as 1500 million and graphical representation is done by Scilab software. It is concluded that the behaviour obtained from the exponential as a well logistic model with the actual population was similar in nature. For example, in 1981 the actual population was 683.3 and the population obtained from the exponential model was 684.25, which is similar to the population obtained from the logistic model.

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