Mueller Matrix Polararimetry Characterization Of Polarization Parameters Of Anisotropic Material

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Abstract: Polarimetric imaging methods take advantage of an object's polarisation properties to produce a more selective presentation. Every pixel of the image can be extended using this method as a combination of separate Mueller matrix entries, with polarisation states serving as the characteristic coefficients. Thus, a high contrast image can be obtained by measuring the polarisation characteristics. In order to comprehend how the polarisation characteristics of light change across an anisotropic wood sample, ellipsometry is a technique that is used. The acquired optical signature matrix aids in comprehending and assessing the sample wood's qualities and imparts knowledge regarding how the wood substance works. By using a sophisticated technique of analysing each pixel individually, along with the sample's normalised iconic matrix technique of analysing each pixel individually, along with the sample's normalised iconic matrix, settings to produce a recognisable Mueller Matrix are addressed and used to identify the polarisation variation of the wood sample. The light we used for this project has a wavelength of 632.8 nm. The findings demonstrate the great potential of this technology for polarisation parameter measurements on anisotropic wood samples.

Keywords: MMI; Optical signature; Ellipsometry, Polarisation, and anisotropy.

Introduction:

A Mueller matrix polarimeter is particularly appealing in the industrial measurement sector and laboratory studies since it provides all of the information concerning the polarisation features of a medium except the overall phase. The Mueller polarimetry method is a popular polarisation analytical method that receives a linear variation between both the polarisation state of incident and excited beams from a sample. This method describes the polarisation properties of the sample by utilising a Polarization State Generator (PSG) and a Polarization State Analyzer (PSA) that have rotating wave plates, a polarizer, and an analyzer. In order to explain the polarisation effects that occur in the sample of interest, photographs were taken with predetermined polarisation states. The Mueller polarimetry method is a popular polarisation measurement technique that obtains a linear variation between the polarisation
state of incident and excited beams from a sample. This method describes the polarisation properties of the sample by employing a Polarization State Generator (PSG) and a Polarization State Analyzer (PSA) that have rotating wave plates, a polarizer, and an analyzer. In order to explain the polarisation effects that occur in the sample of interest, photographs were taken with predetermined polarisation states. After this, the photographs are subjected to additional processing in order to gain a better understanding of the distinguishing change in the image, which is associated with the variation in sample composition. The Mueller matrix measurement system was described in a few of the papers written by researchers. [1] - [7] The 49 images that were collected are used to investigate the optical polarisation effects that occurred simultaneously in the sample, and the results of this investigation are reduced to 16 element Mueller matrix images. After that, the values of these elements are normalised relative to the value of the first element, and the Mat Lab programmes that are used to process images pixel by pixel are used to generate a signature matrix.

**THEORY**

The polarisation state of light can be characterised by four objective factors that are collectively referred to as the Stoke parameters. The column vector is the most common representation of this 4-stroke vector.

\[
S = \begin{pmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{pmatrix}
\]  

(1)

The principle of optical equivalence developed by G.G. Stokes in 1853 demonstrates that the Stokes vector is an exhaustive representation of the polarisation state of a light beam. The Stokes vector (S) of a light beam goes through a linear transformation to become a new Stokes vector (S') whenever the Stokes vector of the light beam is modified (scattered) by an optical element. This transformation is typically referred to as the Mueller or Polarization matrix (M), and its representation is a four-by-four matrix with the common notation.

\[
S' = M \times S
\]  

(2)

\[
\begin{pmatrix}
S'_0 \\
S'_1 \\
S'_2 \\
S'_3
\end{pmatrix} = \begin{pmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34} \\
m_{41} & m_{42} & m_{43} & m_{44}
\end{pmatrix} \begin{pmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{pmatrix}
\]  

(3)

Since the Mueller matrix stores all polarisation information, the Stokes vector used to describe the light beam’s polarisation is sufficient.

The Mueller matrix M, in addition to its standard form, can be represented as

\[
M = m_{11} \begin{pmatrix}
1 & \hat{D}_T^T \\
\hat{P} & m
\end{pmatrix}
\]  

(4)

Where \( \hat{D}_T = \frac{1}{m_{11}}(m_{12} \quad m_{13} \quad m_{14}) \) & \( \hat{P} = \frac{1}{m_{11}}(m_{21} \quad m_{31} \quad m_{41}) \) called as Diattenuation and Polarization vectors correspondingly and ‘m’ is a 3 X 3 matrix.

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Hans Mueller was the first person to use matrices to perform complicated computations in order to solve complicated polarisation problems. In fact, the formalism that bears his name, the Mueller Matrix formalism, was named after him. In order to make significant progress in completing the difficult computations, he computed the matrices for the polarizer, wave plate, and rotator. This was an important step. The Mueller Matrix provides a comprehensive characterization of polarisation elements [11]. Hans Mueller, the person who formalised polarisation calculations based on intensity, is recognised as the namesake of the Mueller formalism. It's possible that not all Mueller Matrices can be realised in the physical world. For a Mueller Matrix to be considered physically realisable, the primary requirement is that the incident Stokes vector must be physically realisable from the resultant Stokes vector through the use of the Mueller Matrix. Because of this, it is necessary to have a Degree of Polarization that is either less than or equal to one, also known as.

\[ P = \frac{\sqrt{(S_1^2 + S_2^2 + S_3^2)}}{S_0} \leq 1 \]  

(5)

The inequality [16] is a well-known restriction on the Mueller Matrix.

\[ (MM^T)^T = \sum_{i,j=0}^{3} m_{ij}^2 \leq 4m_{11}^2 \]  

(6)

The diattenuation is defined as,

\[ D = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}} + T_{\text{min}}} \]  

and values varies from 0 - 1.

The diattenuation of the Mueller Matrix is

\[ D = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}} + T_{\text{min}}} = \frac{1}{m_{11}} \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \]  

(8)

The individual terms constituting the diattenuation vector are operationally defined by

\[ D_H = \frac{T_H - T_V}{T_H + T_V} = \frac{m_{12}}{m_{11}}, \quad D_{45} = \frac{T_{45} - T_{135}}{T_{45} + T_{135}} = \frac{m_{13}}{m_{11}} \quad \text{and} \quad D_C = \frac{T_R - T_L}{T_R + T_L} = \frac{m_{14}}{m_{11}} \]  

(9)

here \( T_H \) is the horizontally polarized light transmittance, \( T_V \) is the vertically polarised light transmittance, \( T_{45} \) - linear 45° polarised light transmittance, \( T_{135} \) - linear 135° polarised light transmittance, \( T_R \) - right circularly polarised light transmittance, and \( T_L \) - left circularly polarised light transmittance.

Polarizance refers to the polarisation that occurs when totally unpolarized light is converted into polarised light.

\[ P = \frac{1}{m_{11}} \sqrt{m_{21}^2 + m_{31}^2 + m_{41}^2} \]  

(10)

and can take values from 0 to 1.

The fast axis and the retardance vector are denoted by
\[ \bar{R} \equiv \bar{R} = \begin{pmatrix} R_{a1} \\ R_{a2} \\ R_{a3} \end{pmatrix} \equiv \begin{pmatrix} R_H \\ R_{45} \\ R_C \end{pmatrix} \quad (11) \]

where the components define the circular, horizontal, and 45° linear retardances. The linear retardance that is still present is

\[ R_L = \sqrt{R_H^2 + R_{45}^2} \quad (12) \]

and the total retardance is

\[ R = \sqrt{R_L^2 + R_{45}^2 + R_C^2} = \sqrt{R_L^2 + R_C^2} = |\bar{R}| \quad (13) \]

The following is a normalised Mueller Matrix \( M \):

\[ M = \begin{pmatrix} 1 & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} = \begin{pmatrix} 1 \\ \bar{D}^T \\ \bar{P} \\ m \end{pmatrix} \quad (14) \]

Where the sub Matrix \( m \) is

\[ m = \begin{pmatrix} m_{22} & m_{23} & m_{24} \\ m_{32} & m_{33} & m_{34} \\ m_{42} & m_{43} & m_{44} \end{pmatrix} \quad (15) \]

And \( \bar{D}, \bar{P} \): diattenuation, polarization vectors and the diattenuator \( M_D \) is considered after the first row of \( M \), and \( M_D^{-1} \) be multiplied by \( M \) to find retarder Matrix \( M_R = MM_D^{-1} \). Then diattenuator of Matrix is specified as

\[ M_D = \begin{pmatrix} 1 & \bar{D}^T \\ \bar{P} & m_D \end{pmatrix} \quad (16) \]

where

\[ m_D = aI_3 + b(\bar{D}.\bar{D}^T) \quad (17) \]

and where \( I_3 \) is the identity matrix for a 3 by 3 grid, and \( a \) and \( b \) are scalars that are obtained from the norm of the diattenuation vector, i.e., where \( I_3 \) is the norm of the diattenuation vector.

\[ D = |\bar{D}|, \quad a = \sqrt{1 - D^2} \quad \text{and} \quad b = \frac{1-\sqrt{1-D^2}}{D^2} \quad (18) \]

The attenuation vector is used to characterise the intensity transmission of the polarization element. The polarization state that is produced as a result of an unpolarized input state may be described using the depolarization vector. This study goes through a few different topics, including attenuation, depolarization, and retardance.

**EXPERIMENTAL PROCEDURE**

To illuminate the sample with predetermined polarisation states, a set of linear Polarizer \( P \) and Quarter wave plate \( Q1 \) was employed. Another pair of linear analyzers \( A \) & quarter
wave plates (Q2) were employed to study the polarization state of light reflected from surface of the sample onto a detector. While measuring, the Analyser and collection optics were held at a scattering angle of 45° from the beam direction. In this method produced 49 photos by spinning the Polarizer and Analyzer appropriately to acquire the 4 X 4 Mueller matrix images. The wood sample was irradiated with a laser with a power of 20mW and a wavelength of 632.8 nm with a specified polarisation state. Throughout the experiment, the group optics are kept at 45° after the direction of the input beam. As seen in Figure 1.

**Figure1: Experimental setup for getting Mueller matrix elements**

The cajanus cajan wood material produced from natural source sample-3 is the item that is now being scrutinised for analysis. The substance that was generated went through a yearlong process of drying in order to allow the moisture content to naturally evaporate without affecting the chemical makeup. After that, the sample was sent to Vitro labs in Hyderabad, India, so that it could be analysed chemically; the results of that examination are presented in Table 1. After receiving a high-quality polishing, the sample now has an average thickness of 1.15 millimetres, width of 58.25 millimetres, and length of 136.45 millimetres.

<table>
<thead>
<tr>
<th>Chemical Composition of Cajanus Cajan</th>
</tr>
</thead>
<tbody>
<tr>
<td>C %</td>
</tr>
<tr>
<td>56.99</td>
</tr>
</tbody>
</table>

Table 1:

The intensity of the photons [14, 15] was measured by directing a stream of photons emanating from the source and passing through a PSG and onto the sample material, then redirecting the reflected beam through a PSA to a CCD detector coupled to a computer. In total, 49 intensity pictures were captured by adjusting various optical elements [16,17] in the PSG and PSA.

The sample's unique optical signature can be obtained from the Mueller matrix. For the 16 Mueller matrix elements, you'll need the 49 intensity images taken at different Polarizer and Analyzer angles. Following the acquisition of 49 intensity pictures, the 16 elemental Muller matrix images [18] may be produced, and their definitions are as follows:

\[
m_{11} = I_{OO} \quad m_{21} = I_{OH} - I_{OV} \quad m_{31} = I_{OP} - I_{OM} \quad m_{41} = I_{OL} - I_{OR}
\]
where \( H \) denotes horizontal polarisation, \( V \) denotes vertical polarisation, \( P \) denotes +45°, \( M \) denotes -45°, \( R \) denotes right circular polarisation, and \( L \) denotes left circular states of polarisation; the first subscript in the intensity parameters denotes the input state, and the second subscript the output state. Figure 4 displays the equivalent pictures, which were acquired with the help of a custom-written MATLAB programme that cropped the images to have identical pixel dimensions and extracted intensity data from each individual pixel.

After obtaining all 16 photos, they are processed once again with a custom MATLAB software that calculates the intensity component of each pixel in order to get the image’s overall intensity information. Because of this transformation, the resulting Muller matrix has a \( m_{11} \) component, which simplifies the analysis and allows for the separation of intensity-dependent effects from the polarisation effects.

**About the wood sample (Cajanus Cajan):**
To be more specific, the Papilionaceae or Fabaceae family. Karnataka, Bihar, Andra Pradesh, Maharastra, Uttar Pradesh, Madhya Pradesh, and are the most common states where this crop is cultivated as a pulse. Pigeon pea, often called red gramme, is a kind of bean native to India. Aadacki, Tuvari, Tuvara, and Shanupushpikaa are all terms used in Ayurvedic medicine. "Arhar" in Unani. Siddha and Tamil both have the word Thuvarai. Purpose: Lowering cholesterol, green leaves are recommended. Cholesterol and phospholipids are reduced, as seen in the pulse (reported to cause flatulence). To treat jaundice, consume a salt-and-water-based leaf paste first thing in the morning. Leaves are applied topically to cure measles and other rashes, as well as for oral disorders. The Indian Ayurvedic Pharmacopoeia recommended the seed for lipid problems and obesity, and the root for its cleansing effects on the blood. Analysis of the amino acids in the seed extract revealed the phenylalanine (26.3% in the entire amino acids) accounts for almost 70% of the extract’s anti-sickling efficacy [19].

**RESULTS & DISCUSSIONS**

The 49 intensity pictures that were collected are utilised to produce the 16 elemental Mueller matrices by utilising the formulae that were presented before. These images are presented in Figure 2 and may be seen there.
Figure 2. Images of a wood sample using the Mueller Matrix

Following the capture of sixteen photos, those images are then analysed here on MATLAB platform in order to extract the evaluations of every single pixel contained within the image. This Mueller matrix has been normalised to the first element of a matrix in order to both simplify the analysis and isolate the intensity-dependent effects that can be seen in the picture. Table 2 is a listing of the elements that make up the Mueller matrix.

<table>
<thead>
<tr>
<th></th>
<th>-0.21642</th>
<th>0.039956</th>
<th>0.055101</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.29999</td>
<td>0.063876</td>
<td>-0.05593</td>
<td>0.011101</td>
</tr>
<tr>
<td>-0.04994</td>
<td>-0.0899</td>
<td>-0.24526</td>
<td>-0.03126</td>
</tr>
<tr>
<td>0.019989</td>
<td>0.15786</td>
<td>-0.12986</td>
<td>-0.06532</td>
</tr>
</tbody>
</table>

Table 2: Elements of the Mueller matrix contained in the sample

Acquiring pictures of diattenuation, retardation, and depolarization after the restrained Mueller matrix yields the results seen in figures 3-5. Table 3 displays the typical diattenuation and depolarization values of the sample.

<table>
<thead>
<tr>
<th>Sample name</th>
<th>Diattenuation</th>
<th>Depolarization</th>
<th>Retardance</th>
</tr>
</thead>
<tbody>
<tr>
<td>cajanus cajan</td>
<td>0.2345</td>
<td>0.8169</td>
<td>2.0621</td>
</tr>
</tbody>
</table>

Table 3. The average values of the parameters diattenuation, depolarization, and retardance

![Fig 3: Depolarisation of the sample](image-url)
CONCLUSIONS

Because of the ways in which we conducted our experiments, we were able to extract an optical fingerprint of the wood specimen in the type of a Mueller matrix. As was to be expected, the wood sample exhibited polarisation anisotropic feature. The tabular representation of the outcomes of our trials reveals this to be true. It is evident, in both the images that have been acquired and from the values that are showcased in the table, that the criterion that is required and sufficient for obtaining Complete/Mueller Polarimeter polarisation has been comfortable. This can be seen both from the images that were obtained as well as from the values that are displayed in the table. Processing the pictures produced pixel by pixel in order to detect the different optical polarisation shifts and scattered intensity distribution that were found in the sample was successful, according to the findings of the tests that were done for this study. These findings were interpreted in terms of the pictures and values that were acquired for the parameters of Diattenuation, Depolarization, and Retardance.

References:


