Designing An Interval Observer For Detecting A Branch Over A Pipeline

*Cortés Cruz Eduardo, **Jorge Sofrony Esmeral, ***Jesús David Aviles Velazquéz

* Universidad Nacional de Colombia, Bogotá, Colombia-11321
** Universidad Nacional de Colombia, Bogotá, Colombia-11321
*** Universidad Autónoma de Baja California - UABC, Baja California, México

Abstract—This article proposes a linear interval observer for a nonlinear system that represents the dynamic system of a fluid that circulates through a pipeline. The method of interval estimation is for defining the difference between the inlet flow rate: \( Q_{in} \) and the outlet flow rate: \( Q_{out} \) when on the pipeline exists a unique branch.

The proposed method is through the design of a linear Luenberger-type observer interval. The interval estimation method considers also all the uncertainties and noises associated to the instrument lectures that involve the numerical values to the inlet flow rate \( Q_{in} \) and outlet flow rate \( Q_{out} \) the upstream pressure head \( H_{in} \) and the downstream pressure head \( H_{out} \). The results show how the proposed interval observer estimates the states taking into account the branch presence over the pipeline.

Index Terms—Interval Arithmetic, Metzler matrix, positive systems, interval observers, pipelines.

I. INTRODUCTION

Interval observers are typically designed to characterize the widest interval that bounds the states of a system when they are susceptible to uncertainties. In this work, we propose an interval observer design using the method exposed by Filippo Cacace and Manes (2015). The interval observer is designed for a dynamic system which has two types of uncertainties, the first one, associated to the perturbations that are present with the input signals, and the second one, associated to the noise measurements in the flow rate indicators. An interval observer is defined as a pair of observers whose dynamics and initial conditions are defined such that their trajectories characterize upper and lower
bounds on the state values at any given instant. The upper observer is designed using values of the uncertain parameters such that its estimates bound the true states from above at any given instant. At the same time the lower observer is designed using values of the uncertain parameters such that its estimates bound the true states from below at any given instant (Chambon et al. (2016)). In this sense, both observers define bounds within the true state values of the system are contained. This condition is helpful when a dynamic system has uncertainties that cannot be completely described but can be defined inside interval whose extreme uncertainty values are represented as \( w(t) = [\underline{w}, \overline{w}] \), where \( \underline{w} \) and \( \overline{w} \) are the extreme values known for the uncertainties represented by the uncertainty vector \( w(t) \). In this regard, in this article we extend the results in Torres et al. (2019) in order to design an interval observer that allows the leak and branch detection in pipelines, considering additional parameters such as the uncertainties associated to the sensor errors that involves the inlet flow rate \( Q_{in} \) and outlet flow rate \( Q_{out} \) measurements, and also the errors associated to the upstream pressure head \( H_{in} \) and the downstream pressure head \( H_{out} \).

The remainder of this article is organized as follows. In Section 2, a general review about the interval arithmetic and the main properties for defining interval vectors and interval matrices is given. Additionally, in this section we include topics about the Metzler matrices and positive systems definitions. In Section 3, we describe the model exposed in Torres et al. (2019) along with the dynamic system that represents the movement of a fluid inside a pipeline with a branch. In this part, the dynamic system is linearized and represented as state variables including the disturbances and noises associated to the state model. In Section 4, we describe the design of the proposed interval observer along with the analysis of the obtained results. Finally, in Section 5 we present the conclusions and future works.

2 PRELIMINARIES

2.1 General Overview of Interval Arithmetic

In this section we expose the mathematical concepts necessary for the design of the proposed interval observer.

Interval Representation. Let \( x \) be a number \( \in \mathbb{R} \) that represents a bounded value in the interval of real numbers \( \mathbb{R} \). For this purpose, \( x \) will be bounded between a lower and an upper limit, respectively as

\[
x' = [x, \bar{x}]
\]

where

\[
x' \in \mathbb{R}^n, \quad x, \bar{x} \text{ are the minimum and maximum numerical values respectively of } x.
\]

Internal Operations for an Interval. These represent the operations that can be performed between the lower and upper limits of an interval, are the following:

- Mean value or nominal value, expressed as:
\[ \text{mid}(x) = \frac{x + \bar{x}}{2} \quad (1) \]

- Interval width, defined as:
  \[ w = \bar{x} - x \quad (2) \]

- Interval radius, defined as the tolerance of the mean value or nominal value:
  \[ \text{rad}(x) = \frac{\bar{x} - x}{2} \quad (3) \]

- Absolute value, defined as the maximum numerical value of an interval:
  \[ \text{abs} (x^I) = \max\{ |x|, |\bar{x}| \} \quad (4) \]

Interval Vector. According to Moore (1988), an interval vector or box, is a set of n-dimensional intervals where each component is exposed as a closed and bounded subset defined as the dot product of n intervals. This is expressed:

\[ [x] = ([x_1^I] \times [x_2^I] \times \cdots \times [x_n^I]) \]

An interval vector of \( \mathbb{R}^n \) is an n-dimensional hyper-rectangle, that is a domain delimited by 2n hyper-planes of dimension 2n – 1, parallel to the axes.

For example, the interval vectors in \( \mathbb{R}^2 \) are rectangles, while the interval vectors in \( \mathbb{R}^3 \) are prisms. Interval vectors can be represented in a compact form:

\[ [x] = [x, \bar{x}] \]

where the interval vector \( x \) is defined as the extreme lower limit, and the interval vector \( \bar{x} \) is defined as the extreme upper limit of the interval vector:

\[ [x] = [x_1, x_2, x_3, \ldots, x_n] \]

\[ [\bar{x}] = [\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_n] \]

The set of all interval vectors in \( \mathbb{R}^n \) is denoted as:
\[ I^{n \times n} = \{|x_1|, |x_2|, ..., |x_n| \}/|x_1|, |x_2|, ..., |x_n| \in I^\mathbb{R} \]

Interval Matrix. It is a matrix whose elements are interval numbers. In fact, the interval representation of each interval component can be expressed as:

\[ [a_{ij}, \bar{a}_{ij}] \in A^I \]

where

\[ a_{ij} \leq \bar{a}_{ij} \]

According with the concepts exposed above the \( A^I \) matrix can be expressed as

\[ A^I \in [A, \bar{A}] \]

This means that an interval matrix is an array where some or all of its components can be made up of intervals (called interval matrix inputs) arranged in rows (or lines) and columns, where a row is each horizontal interval of the interval matrix, and a column is each vertical interval of the interval matrix. An interval matrix is analytically expressed as

\[ A^I \in I^{n \times n} \]

Another way for representing an interval matrix is using the center matrix or nominal matrix and the radius matrix, which are given respectively by the following expressions:

\[ A_c = \frac{(A + \bar{A})}{2} \]

\[ \Delta_r = \frac{(A - \bar{A})}{2} \]

so, the interval matrix can also be represented as:

\[ A^I = [A_c - \Delta_r, A_c + \Delta_r] \]

where the radius matrix \( \Delta_r \) is the part of the matrix that is analytically related to the average interval matrix perturbation.
Metzler Matrix. A square matrix $A \in \mathbb{R}^{n \times n}$ is said to be a Metzler matrix if all its on-diagonal components are less than zero, that is $a_{i,j} < 0$, $\forall i = j$, and all the off-diagonal components are equal to or greater than zero, i.e., $a_{i,j} \geq 0$, $\forall i \neq j$.

2.2 Positive Systems

Given a continuous-time linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t)$$

this continuous system is positive if the A matrix is Metzler, the B and C matrices are non-negatives, the input vector $u(t) \geq 0$, and the initial conditions vector $x(0) \geq 0$ (Cacace et al. (2012)).

A bounded vector $x^I \in I\mathbb{R}^n$ is represented in general form as

$$x^I = [x, \bar{x}]$$

For any set vector $x, \bar{x} \in I\mathbb{R}^n$ has a positive representation given by (Cacace et al-2012) and (Farina / Rinaldi (2011) as:

$$\begin{bmatrix} x^+ = \max(x, 0) \\ x^- = \max(-x, 0) \end{bmatrix} \quad \begin{bmatrix} \bar{x}^+ = \max(\bar{x}, 0) \\ \bar{x}^- = \max(-\bar{x}, 0) \end{bmatrix}$$

Where $x, \bar{x} \in I\mathbb{R}^{2n}$. In this sense, the positive interval representation is given by

$$\begin{bmatrix} x^+(0) \\ x^-(0) \end{bmatrix} \quad \begin{bmatrix} \bar{x}^+(0) \\ \bar{x}^-(0) \end{bmatrix}$$

for the upper limit. The set of expressions shown above apply to any interval vector.

In order to describe the positive representation of any matrix of any dimensions, let us consider a matrix $Q \in I\mathbb{R}^{m \times n}$where the $Q \in Q^I$ and $Q^I = [Q, \bar{Q}]$.

The positive representation for $Q^I$ is given by
\[ Q \in \mathbb{I}_\mathbb{R}^{2m \times 2n} = \begin{bmatrix} Q^+ & Q^- \\ Q^- & Q^+ \end{bmatrix} \]

\[ \overline{Q} \in \mathbb{I}_\mathbb{R}^{2m \times 2n} = \begin{bmatrix} Q^+ & \overline{Q}^- \\ \overline{Q}^- & Q^+ \end{bmatrix} \]

According with the exposed, now we will consider the following system:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + B_d \nu(t) \\
y(t) &= Cx(t) + N\eta(t)
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\) is the state vector of the dynamic system, \(u(t) \in \mathbb{R}^m\) is the input vector, \(y(t) \in \mathbb{R}^q\) is the output vector, \(\nu(t) \in \mathbb{R}^{n_d}\) is the disturbance vector, \(\eta(t) \in \mathbb{R}^{q_n}\) is the noise vector, and the \(A, B, C, B_d, N\), are matrices of adequate dimensions. It is important to clarify that the vectors: \(\nu(t), \eta(t)\) are vector of known values in its extremes.

According with the exposed above, the positive representation of the \(A\) matrix is given by:

\[
A_{kt} = \begin{bmatrix}
M_k - d_k & d_k \\
(M_k - d_k) & M_k - d_k
\end{bmatrix}
\]

Also according with (5), \(M_k\) is the Metzler representation of the \(A\) matrix. \(d_k\) is a diagonal matrix and its components are from the diagonal \(M_k\) matrix.

\[
d_k = \begin{bmatrix}
m_{1,1} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & m_{n,n}
\end{bmatrix}
\]

for more details about the obtained expression in (5) see Filippo Cacace and Manes (2015).

3 ANALYTIC MODEL REPRESENTATION

In this section, we describe the results from Torres et al. (2019) that have constituted the basis for our design. Initially, let us consider the pipeline model given in Figure 1, whose nonlinear model is given by
Figure 1. A free-leak pipeline with a branch (Torres et al., 2019).

\[
\dot{Q}_{in} = \frac{\theta}{z_b - z_0}(H_{in} - H_b) - \alpha Q_{in}^{\gamma+1}
\]

\[
\dot{Q}_{out} = \frac{\theta}{L - z_b}(H_b - H_{out}) - \alpha Q_{out}^{\gamma+1}
\]

Equation (6) shows the parameters related to the friction losses and can be associated to physical parameters of the pipeline and fluid such as the flow rate in the pipeline section (Q), the pressure head loss through H, the gravity acceleration (g), and the cross-sectional area (Ar). Additionally, \( \gamma, \alpha \leq 1 \) and \( \theta = gAr \). (See Torres et al. (2019) for additional details). The nonlinear model in (6) is linearized using the linearization method around a fixed point, and exposed as a state representation in the following form:

\[
\begin{bmatrix}
\dot{Q}_{in} \\
\dot{Q}_{out}
\end{bmatrix} = [A] \begin{bmatrix}
Q_{in} \\
Q_{out}
\end{bmatrix} + [B] \begin{bmatrix}
H_{in} \\
H_b \\
H_{out}
\end{bmatrix}
\]

\[
y(t) = [C] \begin{bmatrix}
Q_{in} \\
Q_{out}
\end{bmatrix} + [D] \begin{bmatrix}
H_{in} \\
H_b \\
H_{out}
\end{bmatrix}
\]

where the state matrices A, B, C and D obtained from the linealization method are:

\[
A = \begin{bmatrix}
-\alpha(\gamma + 1)\bar{Q}_{in}^{\gamma} & 0 \\
0 & -\alpha(\gamma + 1)\bar{Q}_{out}^{\gamma}
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{\theta}{z_b - z_0} & -\frac{\theta}{z_b - z_0} & 0 \\
0 & \frac{\theta}{\theta} & -\frac{\theta}{L - z_b} \\
0 & \frac{L - z_b}{L - z_b}
\end{bmatrix}
\]

(8)
Now, including perturbations and noises associated to the errors in (8), we obtain the dynamic system:

\[
\begin{bmatrix}
\dot{Q}_{in} \\
\dot{Q}_{out}
\end{bmatrix} =
\begin{bmatrix}
A & B_d
\end{bmatrix}
\begin{bmatrix}
Q_{in} \\
Q_{out}
\end{bmatrix} +
\begin{bmatrix}
H_{in} \\
H_{out}
\end{bmatrix} + [B_d] \nu(t)
\]

(9)

\[
\begin{bmatrix}
Q_{in} \\
Q_{out}
\end{bmatrix} =
\begin{bmatrix}
C & B
\end{bmatrix}
\begin{bmatrix}
Q_{in} \\
Q_{out}
\end{bmatrix} + [B] [H_{in}] + [N] [\eta(t)]
\]

With

\[
B_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

(10)

\[
\nu(t) = \begin{bmatrix} 0.01(0.01\sin(20t)) \\ 0.01(0.01\sin(20t)) \end{bmatrix}
\]

\[
\eta(t) = \begin{bmatrix} 0.01\text{rand}(0.01\sin(20t)) \\ 0.01\text{rand}(0.01\sin(20t)) \end{bmatrix}
\]

These matrices and vectors are associated to the perturbations and noises that involve the measurement errors for the upstream / downstream pressure head, and the inlet / outlet flow rate, respectively. It is important clarify that the \(H_{in}, H_{b}\) and \(H_{out}\) values are obtained using measurement instruments installed along the pipeline as has been described in (Torres et al., 2019).

4 INTERVAL OBSERVER DESIGN

In this section we describe the design of an interval observer for the dynamic system exposed in (7) where the disturbances and noises are associated to reading errors in the flowmeters \(\nu(t)\), and signal noises \(\eta(t)\) associated to errors of indicators in the pressure gauges. According to the exposed, we use the design method presented in Filippo Cacace and Manes (2015), in which the characteristic equations are given by.
\[ A_K = \begin{bmatrix} d^k + (A_k - d^k)^+ & (A_k - d^k)^- \\ (A_k - d^k)^- & d^k + (A_k - d^k)^+ \end{bmatrix} \] \hspace{1cm} (11)

\[ \bar{B} = \begin{bmatrix} B^+ \\ B^- \end{bmatrix} \] \hspace{1cm} (12)

\[ \bar{C} = \begin{bmatrix} C^+ \\ C^- \end{bmatrix} \] \hspace{1cm} (13)

\[ \bar{D} = \begin{bmatrix} D^+ \\ D^- \end{bmatrix} \] \hspace{1cm} (14)

About the disturbance and noise vectors, theses were also represented as positive form, taking into account the procedure defined in the section 2 for interval vectors.

From (11) for the \( A_k \in \mathbb{R}^{n \times n} \) matrix, complies the Metzler matrix condition, being \( A_k = A - LC \), where the L gain of observer is obtained by the pole placement method and complies also the Hurwitz condition; however when the matrix \( A_k \) is represented as positive representation: \( A_k \in \mathbb{R}^{2n \times 2n} \), with this representation the \( A_k \) matrix is not Hurwitz.. Therefore, we apply the coordinate transformation method presented by Filippo Cacace and Manes (2015) for designing and to implement the proposed interval observer.

The numerical results of the transformed matrices are:

\[
\tilde{A}_K = \begin{bmatrix} -3.79 & 0.00 & -0.00 & -0.00 \\ 0.00 & -1.49 & -0.00 & -0.00 \\ -0.00 & -0.00 & -3.79 & 0.00 \\ -0.00 & -0.00 & 0.00 & -1.49 \end{bmatrix}
\]

\[
\bar{B}_K = \begin{bmatrix} 1e - 3 & 0 & 0 & 0 & 1e - 3 & 0 \\ 0 & 4e - 4 & 0 & 0 & 0 & 4e - 4 \\ 0 & 4e - 4 & 0 & 1e - 3 & 0 & 0 \\ 0 & 1e - 3 & 4e - 4 & 0 & 4e - 4 & 0 \end{bmatrix}
\]

\[
\bar{C}_K = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}
\]

\[
\bar{D}_K = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]
Being $\bar{A}_K, \bar{B}_K, \bar{C}_K, \bar{D}_K$, the matrices transformed by the method exposed by Filippo Cacace and Manes (2015).

4.1 Results Analysis

According to the numerical values previously described, we obtain the results shown in the following set of figures

Figure 2. Inlet flow rate from Torres et al. (2019) Vs. Inlet flow rate using the proposed interval observer

The Figure 2 shows a comparison between the experimental measurements taken for the inlet flow rate described in Torres et al. (2019) (green curve), and the inlet flow rate obtained using the proposed interval observer (red and blue curves). We can observe that the estimation from the interval observer is acceptable even in the presence of the uncertainties and noise associated with the model. Additionally, it is noticeable that after 20 seconds the interval observer converges to the estimate of the inlet flow rate shown in Torres et al. (2019).

Figure 3 shows the same comparison of the previous case, but this time over the measurements taken for the outlet flow rate. In this case, we can also observe that the estimation obtained through the proposed interval observer is acceptable even in the presence of the uncertainties and noise associated with the model. The interval observer converges to the estimate of the outlet flow rate given in Torres et al. (2019) after 20 seconds.

Figure 3. Outlet flow rate from Torres et al. (2019) vs. Outlet flow rate using the proposed interval observer.

Finally, in Figure 4 we show a comparison between the outlet flow rate obtained from the experimental measurements (Torres et al., 2019) and the interval estimation of the inlet flow rate. In this case, the provided information is relevant in order to detect especial conditions in a branch of
the pipeline when a threshold defined by the lower and upper bounds is exceeded. In this sense, it is possible to determine when a branch is causing a flow decrease, as the case depicted in Figure 4.

Figure 4. Outlet flow rate from Torres et al. (2019) Vs. Inlet flow rate using the proposed interval observer.

5 CONCLUSIONS

In this work, an interval observer has been designed using the results from Filippo Cacace and Manes (2015) and it was applied from the experimental data of Torres et al. (2019) in order to define the inlet flow rate and the outlet flow rate. The exposed results are satisfactory, because the interval observer is linear and its interval estimation performance reaches each real state. The interval estimation The futures applications for this interval observer is for doing FDI analysis in pipelines taking the branch like a leak or more leaks.

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