Edge Trimagic Graceful Labeling Of Some Star Graphs

J. A. Jose Ezhil¹, M. Regees², T. Shyla Isac Mary³

¹Research scholar, Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, kanyakumari District, Tamil Nadu, India., Reg. No – 18113112092011

²Department of Mathematics, Malankara Catholic College, Mariagiri, Kaliakavilai – 629153, Tamilnadu, India.

³Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, kanyakumari District, Tamil Nadu, India.

Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627 012, Tamil Nadu, India.

ABSTRACT

A (p, q) graph G is called an edge trimagic graceful if there exists a bijection f : V(G) ∪ E(G) → {1, 2, 3, …, p + q} such that for each edge xy in E(G), |f (x) – f (xy) + f (y)| = C₁ or C₂ or C₃, where C₁, C₂ and C₃ are constants. In this paper, we proved that the bistar, double star, triple star graph are edge trimagic graceful graphs.

Key words: Graph, trimagic, graceful, trimagic graceful, bistar, double star, triple star.

AMS Subject Classification: 05C78

1. INTRODUCTION

Labeling of a graph G is an assignment of labels to either the vertices or the edges or both subject to certain conditions. Graph labeling was first introduced in1960’s. Graph labeling are of many types such as magic, bimagic, trimagic, antimagic, graceful, harmonious, equitable, etc. In this paper, we are going to study about trimagic graceful labeling of some ladder family graphs.

Magic labeling was introduced by Sedlacek [1]. In 1970, Kotzig and Rosa [2] defined, an edge magic labeling of graph G is a bijection f: V∪E → {1, 2, ..., p + q} such that, for each edge uv ∈ E(G), f(u) + f(uv) + f(v) is a magic constant. In 2004, Edge Bimagic labeling was introduced by J. B. Babujee [6] as a graph G with a bijection f: V ∪ E → {1, 2, ..., p + q} such that, for each edge uv ∈ E(G), f(u) + f(uv) + f(v) is either k₁ or k₂.
In 2013, C. Jayasekaran, M. Regees and C. Davidraj [6] introduced the edge trimagic total labeling of graphs. An edge trimagic total labeling of a \((p, q)\) graph \(G\) is a bijective function \(f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}\) such that for each edge \(xy \in E(G)\), the value of \(f(x) + f(xy) + f(y)\) is equal to any of the distinct constants \(k_1\) or \(k_2\) or \(k_3\). A \((p, q)\) graph \(G\) is called an edge magic graceful if there exists a bijection \(f: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}\) such that for each edge \(xy \in E(G)\) the value of \(|f(x) + f(y) - f(xy)| = k\), a constant [3]. The graph \(G\) is said to be super edge magic graceful if \(V(G) = \{1, 2, \ldots, p\}\).

A star graph \(S_k\) is a complete bipartite graph \(K_{1,n}\) of order \(n + 1\) is a tree on \(n + 1\) vertices, in a star graph one vertex has degree \(n\) and the remaining \(n\) vertices have degree 1. Stars may also be described as the only connected graphs in which at most one vertex has degree greater than one. \(B_{m,n}\) is a \((m, n)\) Bistar obtained from two disjoint copies of \(K_{1,m}\) by joining the central vertices by an edge. The Double Star \(K_{1,n,n}\) is a tree obtained from the star \(K_{1,n}\) by adding a new pendent edge to each of the existing \(n\) pendent vertices. The Triple Star \(K_{1,n,n,n}\) is a tree obtained from the double star \(K_{1,n,n}\) by adding a new pendent edge to each of the existing \(n\) pendent vertices [7]. We use the Dynamic Survey of Graph Labeling by Joseph A. Gallian [4] for more references. In this paper, we have proved bistar, double star, triple star, splitting graph of a star graph are edge trimagic graceful graphs.

2. MAIN RESULTS

**Theorem 2.1:** The Bistar \(B_{m,n}\) graph admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for all \(n\) and for \(m \geq 2\).

**Proof:** Let the vertex set of the Bistar graph be, \(V(B_{m,n}) = \{u, v, u_i, v_j / 1 \leq i \leq m, 1 \leq j \leq n\}\) and the edge set be \(E(B_{m,n}) = \{uv, uw_i, uu_i, vv_j / 1 \leq i \leq m, 1 \leq j \leq n\}\). Then \(B_{m,n}\) has \(m + n + 2\) vertices and \(m + n + 1\) edges.

Define a bijection \(\varphi: V \cup E \rightarrow \{1, 2, 3, \ldots, 2m + 3n + 2\}\) such that

- \(\varphi(u) = 2m + n + 1\)
- \(\varphi(v) = 2m + n + 2\)
- \(\varphi(u_i) = i, 1 \leq i \leq m\)
- \(\varphi(v_j) = m + j, 1 \leq j \leq n\)
- \(\varphi(uv) = 2m + n + 3\)
- \(\varphi(u_i u_i) = m + n + i, 1 \leq i \leq m\)
- \(\varphi(v_j v_j) = 2m + n + j + 3, 1 \leq j \leq n\)

For each edge \(uv \in E(B_{m,n})\), \(|\varphi(u) - \varphi(uv) + \varphi(v)\|\) will get any one of the constants \(c_1 = |2m + n|\), \(c_2 = |m + 1|\) and \(c_3 = |m - 1|\). Therefore the Bistar graph \(B_{m,n}\) admits an edge trimagic graceful labeling for all \(n\) and \(m \geq 2\). Since the Bistar graph \(B_{m,n}\) has \(m + n + 2\) vertices and these \(m + n + 2\) vertices have labels \(1, 2, \ldots, m + n + 2\) for all \(n\) and \(m \geq 2\), hence the graph \(B_{m,n}\) is a super edge trimagic graceful.

**Example 2.2:** An edge trimagic graceful labeling of \(B_{5,4}\) is given in figure 2.1.
Figure 2.1: B_{5,4} with c_1 = 14, c_2 = 6 and c_3 = 4.

**Theorem 2.3:** The Double Star graph $K_{1,n,n}$ admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for $n > 2$.

**Proof:** The vertex set of the Double Star graph be, $V(K_{1,n,n}) = \{v_0, v_1, v_2, \ldots, v_{2n}\}$ and the edge set be $E(K_{1,n,n}) = \{v_0v_i, v_iv_{i+1} \mid 1 \leq i \leq n\}$. Then the Double Star graph $K_{1,n,n}$ have $2n + 1$ vertices and $2n$ edges.

**Case 1:** $n$ is odd
Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, \ldots, 4n + 1\}$ such that
\[
\varphi(v_0) = 1; \varphi(v_i) = i + 1, 1 \leq i \leq n
\]
\[
\varphi(v_{n+i}) = \begin{cases} 
  n + i + 1, & 1 \leq i \leq \frac{n+1}{2} \\
  n + i + 1, & \frac{n+1}{2} \leq i \leq n 
\end{cases}
\]
\[
\varphi(v_i v_{n+i}) = \begin{cases} 
  3n + 2i, & 1 \leq i \leq \frac{n+1}{2} \\
  2n + 2i, & \frac{n+3}{2} \leq i \leq n 
\end{cases}
\]
and $\varphi(v_0v_1) = 2n + i + 1, 1 \leq i \leq n$
For each edge $uv \in E(K_{1,n,n})$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will get any one of the constants $c_1 = \left|1 - 2n\right|$, $c_2 = \left|2 - 2n\right|$ and $c_3 = \left|2 - n\right|$. Therefore the Double star graph $K_{1,n,n}$ admits an edge trimagic graceful labeling for odd $n > 2$.

**Case 2:** $n$ is even
Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, \ldots, 4n+1\}$ such that
\[
\varphi(v_0) = 1; \varphi(v_i) = i + 1, 1 \leq i \leq n
\]
\[
\varphi(v_{n+i}) = \begin{cases} 
  n + i + 1, & 1 \leq i \leq \frac{n}{2} \\
  n + i + 1, & \frac{n}{2} \leq i \leq n 
\end{cases}
\]
\[
\varphi(v_i v_{n+i}) = \begin{cases} 
  3n + 2i, & 1 \leq i \leq \frac{n}{2} \\
  2n + 2i, & \frac{n+4}{2} \leq i \leq n 
\end{cases}
\]
and $\varphi(v_0v_1) = 2n + i + 1, 1 \leq i \leq n$
Hence for each edge uv ∈ E (K_{1,n,n}), \[ |φ(u) − φ(uv) + φ(v)| \] will get any one of the constants c₁ = |1 − 2n|, c₂ = |2 − 2n| and c₃ = |1 − n|. Therefore the Double star graph K_{1,n,n} admits an edge trimagic graceful labeling for even n ≥ 2.

Since the Double star graph K_{1,n,n} has 2n + 1 vertices and these 2n + 1 vertices have labels 1, 2, ..., 2n + 1 for all n > 2, hence K_{1,n,n} is a super edge trimagic graceful.

**Example 2.4:** An edge trimagic graceful labeling of K_{1,6,6} is given in figure 2.2.

![Figure 2.2: K_{1,6,6}](image)

**Theorem 2.5:** The Triple Star graph K_{1,n,n,n} admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for n ≥ 2.

**Proof:** Let the vertex set of the Triple Star graph be, V (K_{1,n,n,n}) = \{ v₀, v₁, v₂, ..., v₃n \} and the edge set be E (K_{1,n,n,n}) = \{ v₀v₁, v₁vₙ₊₁, vₙ₊₁v₂n₊₁ / 1 ≤ i ≤ n \}. Then the triple star graph K_{1,n,n,n} has 3n + 1 vertices and 3n edges.

Define a bijection φ: V ∪ E → \{ 1, 2, 3, ..., 6n + 1 \} such that

- φ(v₀) = 1
- φ(v₁) = i + 1, 1 ≤ i ≤ n
- φ(vₙ₊₁) = n + i + 1, 1 ≤ i ≤ n
- φ(v₂ₙ₊₁) = 2n + i + 1, 1 ≤ i ≤ n
- φ(v₀v₁) = 3n + i + 1, 1 ≤ i ≤ n
- φ(v₁vₙ₊₁) = 4n + 2i, 1 ≤ i ≤ n
- φ(vₙ₊₁v₂ₙ₊₁) = 4n + 2i + 1, 1 ≤ i ≤ n

For each edge uv ∈ E (K_{1,n,n,n}), \[ |φ(u) − φ(uv) + φ(v)| \] will get any one of the constants c₁ = |1 − 3n|, c₂ = |2 − 3n| and c₃ = |1 − n|. Therefore the Triple star graph K_{1,n,n,n} admits an edge trimagic graceful labeling for n ≥ 2. Since the Triple star graph K_{1,n,n,n} has 2n + 1 vertices and these 2n + 1 vertices have labels 1, 2, ..., 2n + 1 for all n ≥ 2, hence the graph K_{1,n,n,n} is a super edge trimagic graceful.

**Example 2.6:** An edge trimagic graceful labeling of K_{1,5,5,5} is given in figure 2.3.

![Figure 2.3: K_{1,5,5,5}](image)
Figure 2.3: $K_{1,5,5,5}$ with $c_1 = 14$, $c_2 = 13$ and $c_3 = 4$.

REFERENCES


