Performance Analysis Of Channel Coding In A Mobile Wimax System With Reduced Transmission Power

Sabhyata Uppal Soni

Assistant Professor Department of Electrical & Electronics, UIET, Panjab University Chandigarh - 160014 (INDIA)

Abstract- In this paper the use of complementary Golay Sequences (CGS) for peak-to-power ratio (PAPR) reduction and forward error correction (FEC) in an orthogonal frequency division multiplexing (OFDM)-based WIMAX system is analyzed and performance is examined. We study the PAPR reduction performance depending on different QAM schemes. Regarding their error correction capabilities, these sequences are investigated considering M-QAM constellations applied to the OFDM signal specified in IEEE 802.16 standard. High peak-to-average power ratio (PAPR) is the major drawback for multicarrier transmission. In this paper, we propose reliable PAPR reducing method that consists in putting together a Golay sequence with the input sequence. This new method offers competitive performance in terms of low bit error rate (BER) and low PAPR compared to previously published methods. Computer simulation for standard WIMAX in IEEE 802.16e system shows PAPR improvement and also Bit Error Rate (BER) reduction by comparison to the competitive methods. Moreover, the whole system is simulated in MATLAB to validate the proposal in a prototype, where its feasibility has been confirmed.

Keywords: OFDM; Complementary Golay sequences; PAPR reduction; WIMAX.

1. Introduction

Currently needs of bandwidth and flexibility are imposing the use of efficient modulations that are suitable forth characteristics of wireless channels. This is one of the reasons why multicarrier modulation techniques are increasingly used in Worldwide Interoperability for Microwave Access (WIMAX) standards, such as IEEE 802.16a [1]. The choice of multicarrier technique orthogonal frequency division multiplexing (OFDM) is due to its good performance in multipath environments. High PAPR is the major drawback of multicarrier transmission. Various researches have been proposed to reduce this factor. These techniques are divided into three categories. The first one is based on coding techniques such as Reed Muller code, [2] and block code, [3]. These PAPR reduction techniques have shown significant advantages for reduced number of subcarriers N, since the length and the performances of codes are directly related to N. The second is based on adding an extra signal to the data such as Clipping [4], Tone reservation [5] and Active Constellation Extension [6]. The disadvantage of this class of techniques is the undue increase of transmitted power. The third group is based on techniques such as Partial transmit Sequences [7], and Selective Mapping [8]. These techniques have shown significant PAPR enhancements at the cost of high complexity and also bit error rate loss due to side information energy. A promising technique proposed in [9] and [10] is based on the use of Golay sequences as PAPR reduction technique. This technique has attracted considerable interest in terms of PAPR reduction for sequences of significant length. In this paper, we analyze the performance of this method and compare the theoretical gains promised in the literature with those achieved by simulation. The focus in this study is on WiMax systems based on the OFDM mode of the IEEE 802.16e specifications. The performance evaluation is conducted in terms of PAPR reduction and BER at the receiver. The remaining of the paper is organized as follows. In section 2, we introduce OFDM signal, PAPR factor and complementary cumulative distribution function (CCDF) of this
factor. Next, section 3 describes Golay sequences as a PAPR reduction technique. Thereafter, Section 4 shows the physical implementation and the performance of this technique in terms of CCDF and BER and finally section 5 states the conclusion.

I. OFDM SIGNAL & PAPR

OFDM divides the available bandwidth into several orthogonal sub-carriers that can convey either data or control information. OFDM signal is the sum of many independent signals modulated onto sub-channels of equal bandwidth. Let us define N symbols in OFDM as

\[ \{X_n, n = 0, 1, \ldots, N-1\} \]

The complex baseband representation of a multicarrier signal consisting of N subcarriers is given by:

\[ x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f t}, 0 \leq t < T \]

(1)

Where \( j = \sqrt{-1} \), \( \Delta f \) is the subcarrier spacing, and \( NT \) denotes the data block period. In OFDM systems the subcarriers are assumed to be mutually orthogonal (i.e., \( \Delta f = \frac{1}{NT} \))

The PAPR is defined by:

\[ \text{PAPR} = \frac{\max|\{x(t)\}|^2}{E[|x(t)|^2]} \]

(2)

Where \( E[.] \) denotes expectation. For certain OFDM symbols, input data may produce a high-PAR signal. For this reason, it is common in the literature to denote the PAR of the signal as the maximum PAR over the whole transmitted signal, and thus, this is the definition used through the paper. The complementary cumulative distribution function (CCDF) of the PAPR is one of the most commonly used performance measure for PAPR reduction methods. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold. In particular, a baseband OFDM signal with N subcarriers has

\[ \text{PAPR} \leq 10 \log (\text{dB}) \]

(3)

The crest factor (CF) which is widely defined as the square root of the PAPR.

\[ \text{CF} = \sqrt{\text{PAPR}} \]

(4)

From the central limit theorem, it follows that for large values of N (N>64), the real and imaginary values of \( x(t) \) become Gaussian distributed. Therefore the amplitude of the OFDM signal has a Rayleigh distribution, and is given by, [11]:

\[ F(z) = 1 - \exp(z) \]

(5)

The CCDF of the PAPR of a data block with Nyquist rate sampling is defined as:

\[ P(\text{PAPR} > z) = 1 - P(\text{PAPR} \leq z) \]

\[ = 1 - F(z)^N \]

\[ = 1 - (1 - \exp(-z))^N \]

(6)

II. GOLAY SEQUENCES AS A PAPR REDUCTION TECHNIQUE

Golay sequences were initially introduced by Marcel Golay, and they became an efficient method to reduce PAPR in OFDM system, Let \( a = (a_0, a_1, \ldots, a_{n-1}) \) be a sequence of length \( n \) and a characteristic \( H \), such that each entry \( a_i \in Z_H \),

\[ \xi = \exp(2\pi \sqrt{-1}/H), \text{ the periodic autocorrelation of } a \text{ in } u \text{ is given by} : \]
\[ C_d(u) = \sum_{i=0}^{n-1-u} \xi^{ai-ai+u} \quad (7) \]

Any pair of Golay complementary sequences is a pair \((a, b)\) such that the sum of its autocorrelation functions is equal to 0; \(C_d(u) + C_b(u) = 0, \) for \(0 < u < n.\) Each of them is called a Golay complementary sequence or a Golay sequence. The instantaneous power of the signal \(x\) is given by:

\[ P_s(t) = |s(t)|^2 = \sum_{i,j} \xi^{i(t) - j(t) - H(i-j)\Delta f} \quad (8) \]

Let \(x(t) = x_i\) for \(0 \leq \Delta t \leq 1\) and \(j = i + u,\) then, we obtain:

\[ P_s(t) = n + \sum_{u \neq 0} \sum_i \xi^{i(-u)} H_u \Delta ft \quad (9) \]

By using the formula of auto-correlation, the formula of the instantaneous power is given by:

\[ P_s(t) = n + \sum_{u \neq 0} \sum_i C_x(u) \xi^{-Hu \Delta ft} \quad (10) \]

Let \(x = (x_0, x_1, \ldots, x_m)\) and \(y = (y_0, y_1, \ldots, y_{m-1})\) be two code words, then \(x\) and \(y\) are a pair of Golay complementary sequences if \(C_x(u) + C_y(u) = 0\) for \(u \neq 0.\) Using the above definitions, we can conclude that the PAPR of a Golay sequence is at most equal to 2. The proof is evident since the sum of instantaneous power is equal to 2n, and given that the power is positive, so each output is less than 2n then since the average power is equal to n, so the PAPRx = \(\frac{P_x(t)}{n} \leq 2 = 3dB.\)

Assuming that Golay sequences provide reduced PAPR in OFDM system, we now discuss the generation of these sequences. Davis and Jedwab, in [7], gave a general method for generating Golay sequences using Boolean functions. A Boolean function is a function of \(Z_2^m = \{ (x_1, x_2, \ldots, x_m) \mid \in \{0, 1\}\} \) in \(Z_2,\) each variable \(x_i\) is a Boolean function \(f_i(x_1, x_2, \ldots, x_m) = x_i\)

By considering the \(2^m\) monomial \(1, x_1, x_2, \ldots, x_m, x_1x_2, x_1x_3, \ldots, x_{m-1}x_m, \ldots, x_1x_2 \ldots x_m\)

Any Boolean function is a linear combination in \(Z_2\) of these monomials, such that the coefficients of each monomial belong to \(Z_2.\) Davis and Jedwab have specified a sequence \(f\) of length \(2^n\) for each function \(f\) by a list of values given by \(f(x_1, x_2, \ldots, x_m)\) where all \(2^n values\) are in lexicographic order. So, if \((i_1, i_2, \ldots, i_m)\) is the binary representation of \(i = \sum_{j=1}^{m} i_j 2^{mj},\) then \(i^\text{th} \text{ item} f = f(i_1, i_2, \ldots, i_m).\) Using these Boolean functions, Davis and Jedwab established an explicit form for generating a Golay sequence, such that for any permutation \(\pi\) of symbols \((1, 2, \ldots, m)\) and any \(c, c\in Z_2^b a(x_1, x_2, \ldots, x_m) = 2^h-1\sum_{k=1}^{m} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{m} c_k x_k + c\) is a Golay sequence in \(Z_2^b\) of length \(2^m.\) This definition of Golay sequences gives \(2^{h(m+1)}m! / 2\) Golay possible sequences in \(Z_2^b\) of length \(2^m.\)

III. PERFORMANCE OF AN OFDM-BASED WIMAX SYSTEM ENCODED WITH GOLAY SEQUENCES

A new distortion-less technique is proposed to reduce the BER and to overcome the problem of large envelope variation by adding redundancy in the form of sequence in the input data. Combinations of Golay sequence along with the input data results in reduced peaks and lowers BER. The PAPR reduction performance of the proposed schemes was evaluated by performing a series of simulations. The simulation is carried out for \(N = 16, 64, 256\) subcarriers. The performance of an encoded OFDM system by using CGS in terms of PAR reduction and Bit Error Rate (BER) has been studied for a WiMAX environment. Besides the PAR bounding characteristics of the CGS, they also provide FEC due to the correlation of these sequences with Reed Muller codes. Therefore, combining the PAR reduction and FEC capabilities the overall system performance can be increased. Fig 1 shows the basic simulation model of WiMAX-PHY Layer. WiMAX physical layer is based on orthogonal frequency division multiplexing. OFDM is the transmission scheme of choice to enable high-speed
data, video, and multimedia communications and is used by a variety of commercial broadband systems, including DSL, Wi-Fi, Digital Video Broadcast-Handheld (DVB-H), besides WiMAX. The Model consists of three main components namely transmitter, receiver and channel. Transmitter and receiver components consist of channel coding and modulation sub-components whereas channels are modeled as AWGN and fading channels.

![Block diagram of WiMAX system used for simulation](image)

**Fig 1:** Block diagram of WiMAX system used for simulation

The simulation is undertaken by focusing on the WiMAX data frame which employs OFDM format with 256 subcarriers. The detail of WiMAX data frame is described by IEEE 802.16e. The performance of a data transmission system is usually analyzed and measured in terms of the probability of error at given bit rate and SNR. The model is tested for Golay sequence. It is analyzed for 16 QAM, 64 QAM and 256 QAM and the results are shown in figures below. Fig 2 shows the analysis of BER for WiMax system using Golay sequence. Fig 3 shows the same without addition of sequence. It can be seen that BER is reduced by addition of Golay sequence. For comparative analysis we plot both together as in Fig 4. The simulation depicts the comparison of performance of system corresponding to different QAM modulation schemes. In this paper, the results of proposed method are compared with the other methods presented in literatures [12]. For a typical SNR value of 12 dB, the BER values for 256 QAM uncoded and 256 QAM coded modulations are 1.685 and 0.969 respectively viz. the system performance is improved by 22.2 dB. Similarly, system performance is improved by 9.7 dB for SNR of 9. Fig 5 shows the theoretical results.

**Figure 2:** BER vs SNR (dB) without Error correction in 16-QAM 1/2, 64-QAM 3/4, and 256-QAM 3/4 with Guard width ratio (G): 1/4, No of Sub carriers and FFT size 256 on AWGN Wimax model.

**Figure 3:** BER vs SNR (dB) for Effect of Error correction in 16-QAM 1/2, 64-QAM 3/4, and 256-QAM 3/4 with Guard width ratio (G): 1/4, No of Sub carriers and FFT size 256 on AWGN Wimax model
**Figure 4:** Comparison of BER vs SNR (dB) with and without Error Correction in 16-QAM 1/2, 64-QAM 3/4, and 256-QAM 3/4 with Guard width ratio (G): 1/4, No of Sub carriers and FFT size 256 on AWGN Wimax model.

**Figure 5:** Theoretical BER Vs SNR (dB)

**Figure 6:** CCDF for the PAPR obtained without Error correction technique for data subcarriers of WIMAX.

**Figure 7:** CCDF for the PAPR obtained with Error correction technique for data subcarriers of WIMAX.
Figure 7: CCDF Comparison for PAPR obtained without and with Error correction technique for the data subcarriers of WiMAX.

Fig 6 and 7 shows the obvious performance of the proposed method in terms of PAPR.

Fig 6 shows the performance of the proposed system using 16,64 and 256 QAM for uncoded system in terms of CCDF. Fig 7 shows the same results for Golay encoded system. The PAPR for 256 QAM uncoded signal is around 8 dB whereas PAPR for the Golay encoded signal is close to 7 dB. From Fig 8 which shows the comparison of uncoded and coded systems, it can be deduced that, the PAPR can be reduced by nearly 1 dB using the proposed method. Although we theoretically expected a PAR limited to 3 dB after Golay encoding, it was proved in [13] that there is a difference between simulations and implementation of around 3 dB. A theoretical justification may be that the effect of sequence insertion at the frequency domain is to rotate a phase shift for the corresponding time domain waveform. With the phase shift, the OFDM waveform can avoid the coherent summation for different frequency components and thus reduce the PAPR. The proposed Golay encoded OFDM-WIMAX system has been implemented in MATLAB. In all cases the encoded OFDM system obtains lower PAR in several dB than the uncoded system. The results of proposed method can be compared with the theoretical results presented in literatures [14]. The proposed PAP reduction method provides lower PAPR than the system using no PAPR reduction method, for all modulation types.

5. Conclusion

OFDM is a good aspirant to build next generation high bit rate WIMAX but some implementation impairments such as its high peak-to-average power ratio must be solved to avoid the loss in efficiency in high power amplifiers. In this paper, the performance of CGS has been analyzed from the point of view of both PAR reduction and forward error correction. In this study we have investigated the PAPR in the WiMAX system. We investigate the simulation performance of WiMAX OFDM PHY Layer under different modulation schemes and a method is suggested to reduce the PAPR where the simulation results show that, the peak power reduces by about 4 dB. Simulation results also show that the proposed solution can fulfill the bit-error-rate requirements of WiMAX. The probability analyses in the simulations are clear demonstrations providing the improvement work for PAPR reduction.

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