

A Novel Model For Determining Boundary Shape Based On The Optimum Tolerance Value For Polygon Approximation

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Abstract

Polygon approximation is a digital curve method lead to create piecewise of linear segments for digital curve. The automated determination of optimal tolerance value for a shape is a vital stage in many applications such as pattern classification and image understanding applications. This research proposes a novel model for determining the optimum tolerance value that represents the boundary shape. The method relies on the Min-# Polygon approximation criteria. It uses a recursive algorithm in combination with an interpolation model to reproduce a curve or a boundary shape from subset of original points/vertices. Then, the optimum boundary shape has been selected by applying a robust direct curve descriptor. Results indicated that the proposed method accurately detected optimum tolerance value based on minimal dominant points that reproduced the same original boundary shape. The proposed technique produces more accurate and a better identification of the optimal tolerance value of boundary shape generation based on limited points to reproduce a boundary shape leading to reduce the complexity of boundary shape determination.

Keywords

Polygon Approximation, Boundary Shape, Tolerance Value, Elliptic Curve.

Introduction

The Polygon approximation model has emerged as a powerful geometric tool in numerous fields including digital cartography, CAD applications, shape analysis, and image representation (Kalaivani, and Ray, 2019). Polygonal approximation is a digital curve method that uses to create piecewise of linear segments for digital curve. The main challenge of the shape estimation is to reproduce the same shape with limited points and less error. To meet this challenge, different approaches have been introduced for effective polygonal approximation such as (Bellman, 1961) (Stone, 1961) (Montanari, 1970) (Perez, ad Vidal, 1994) (Hosur, and Ma, 1999) (Chau, ad Siu, 2001) (Marji, and Siy, 2003) (Kolesnikov, 2012) (Zhu, and Seneviratne, 1997).

Two types of polygon approximation problems can be categorized: Min- ϵ problem and Min- # problem (Aguilera-Aguilera, et al. 2014). The main challenge to solve the Min- ϵ problem is to reduce the error ϵ of approximation, where the approximated shape segment numbers are given before the approximated task. The main challenge to solve the Min- # problem is to hinder the approximation error ϵ exceeding a given maximum tolerance value. Hence, the aim of each type is to estimate the original curve points P from another polygonal curve vertices Q that is a subset of original points using the polygon shape. Therefore, selection of significant optimal dominant points plays an important role in the shape approximation (Kalaivani, and Ray, 2019) (Aguilera-Aguilera, et al. 2014).

Current methods to solve the above problems can be grouped as metaheuristic approaches, heuristic approaches, and optimal approaches. Approaches in the metaheuristic category include Genetic algorithms (Yin, 1999) (Wang, B. et al. 2009), Colony optimization (Yin, 2007) [14], Particle swarm optimization (Wang, et al. 2008) (Wang, J. et al. 2009) and Tabu Search (Yin, 2007) (Yin, P. Y. 2000). Heuristic approaches provide non-optimal polygonal approximations with less calculation cost. Heuristic approaches can be listed as: Sequential scan approach (Sklansky, and Gonzalez, 1980) (Ray, and Ray, 1994), Split approach, Merge approach and Split-merge approach (Aguilera-Aguilera, et al. 2014). Optimal approaches provide optimal polygonal approximations but consume more computational cost. Generally, optimal methods rely on dynamic programming or an A* search method (Aguilera-Aguilera, et al. 2014) (Salotti, 2001).

Determining the optimal tolerance value to reliably represent boundary shape for polygon approximations is pivotal. Accordingly, this research proposes a new approach that uses Ramer-Douglas-Peucker (RDP) method (Ramer, 1972) (Douglas, and Peucker, 1973) with cubic spline interpolation technique (Smith, 1992) to generate the boundary shapes. Then, Elliptic Fourier Descriptor (EFD) method (Zahn, and Roskies, 1972) (Godefroy, 2012) is employed to detect the optimal boundary shape. This paper is represented as follows.

Section 2 states the related works. Section 3 explains the proposed method. Experimental results are reported in Section 4 next before the main conclusions are drawn from the suggested model.

Related Works

The related works that discussed here are narrowed to the proposed research. They are utilized recently and frequently in many approximation approaches to find optimal solution for shape representation. For example, Aguilera, et al., (Aguilera, et al. 2015) introduced a new technique that optimally solved the min- ϵ problem and found the optimal polygonal approximation of a digital curve applying mixed integer programming algorithm. The algorithm is faster than the dynamic programming technique because it does not require an initial point to be chosen, in contrast to dynamic programming based optimal polygonal approximation algorithm (Perez, ad Vidal, 1994).

A flexible polygonal approximation method proposed by Parvez (Parvez, 2015) was based on relocating the contours which provided minimal approximation error number. This approach indicated that the approximating polygon can lie outside the original contour and these new points are detected as dominant points. The local neighborhood is measured for each contour and the neighborhood that approximated the polygon with minimal error value is added as a new vertex in the approximating polygon that later used as a dominant point. The main objective of the approach is that instead of raising the value of dominant points in the approximating polygon which provided minimal error, vertex relocation is suggested to calculate optimal polygonal approximation. This approach produced flexibility in estimation to limit error value.

Unsupervised polygon approximation algorithm introduced by Madrid-Cuevas et al. (Madrid-Cuevas, et al. 2016) based on convexity/concavity tree method and split-merge technique for the approximation. The algorithm achieved significant results and balanced between the min-# and min- ϵ criteria. It provides a significant merit number with real contour, but it requires more complex computations.

Fernandez-Garcia et al. (Fernandez-Garcia, et al. 2016) applied the modified (RDP) approach to subdivide a curve to perform scale independence. This strategy introduced four different thresholding techniques. The approach is nonparametric and requires several stages to provide a significant approximation.

Backe and Bruno (Backes, and Bruno, 2013) suggested a new approach based on graph theoretic strategy to estimate the polygon. In this approach, each point treated as a vertex in the graph and election of vertices is started using vertex betweenness. The vertex betweenness identifies the rank of each vertex in the graph according to the shortest path

value that passing through it. It requires to select the high transitivity area of the graph, so this strategy is identical to detect the dominant point. To perform this, a developed version of the Bellman–Ford approach (Cherkassky, et al. 1996) is harnessed and the path optimization is applied to achieve a significant estimation with minimal value of vertices. Therefore, this strategy uses the objective function strictly for estimation and consumes modest execution time.

None of the approaches above that investigated the optimal tolerance value that represents most important procedure for approximation task. Also, most of existing methods are based on the split–merge mechanism and require initial point for the approximation measurement.

To the best of the author’s knowledge, this is the first time the optimal tolerance value has been assessed to select the optimal boundary shape based on a novel technique that may be combined in a number of different ways resulting in highly successful adaptive systems.

Methodology

The description of the proposed method for selection the optimal approximation shape from testing database is illustrated in Fig. 1. Each stage in Fig. 1 is explained below.

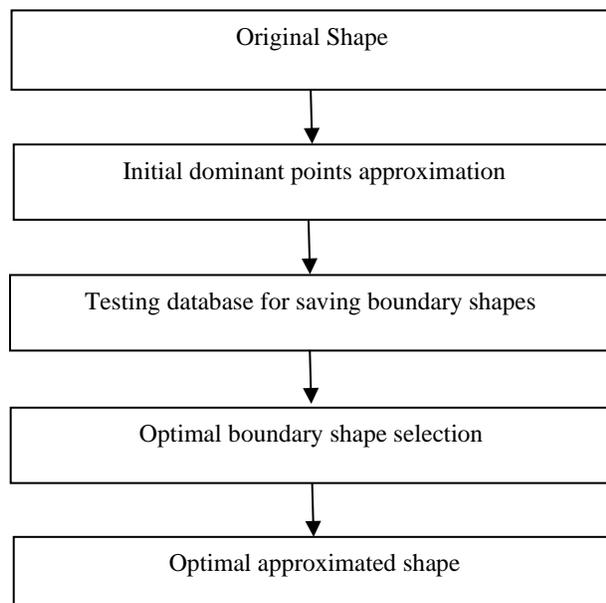


Fig. 1. General workflow of the proposed technique

Ramer-Douglas-Peucker (RDP)

Ramer-Douglas-Peucker (RDP) technique is commonly used in digital curve approximation based on minimal number of vertices (Ehret, and Neuper, 2014) (ZHANG, et al. 2016). It

represents the series points of curve by a subset of original numbers obtained from comparison process for each iteration with predefined tolerance value (ZHANG, et al. 2016).

Where the initial curve is defined by set of points $C_i(x_i: x_i) \in Q$, $i=1 \dots n$ and approximated by piecewise liner function with points $C_j(x_j: x_j) \in Q$, $j=1 \dots m$, $m \leq n$. The algorithm recursively executes for each stage/interval to select the start and end points as key points and applies the comparison process to find the closest value of perpendicular distance to tolerance value ε between the approximated points and original points. If this distance is more than the tolerance value ε then the algorithm subdivides the original interval of the curve as a new interval. After that, it saves the first point and redefines the base point of perpendicular distance as a last point with applying the new comparison process (Melnik, and Viazovskyy, 2016).

Cubic Spline

Curve fitting interpolation is an important tool for data visualization and expectation. It is widely used in many fields including computer aided geometric, computer graphic, and curve design (Du, et al. 2020) (Hammadie, et al. 2019). Cubic spline is one of the common data interpolation models that provides smooth and impressive results (Hammadie, et al. 2019). It formulates as follows, if there are $n+1$ points then the number of intervals and groups of coefficients according to fitting these points is equal to n , where each group is $\{a_i, b_i, c_i, d_i\}$, for $i=1, 2, 3, \dots, n-1$.

The fitting equation and coefficients can be defined by S_i equations below (Smith, 1992).

$$Y_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3 \quad (1)$$

$$d_i = y_i \quad (2)$$

$$b_i = \frac{s_i}{2} \quad (3)$$

$$a_i = \frac{s_{i+1} - s_i}{h_i} \quad (4)$$

$$c_i = \frac{2h_{i+1} + h_i s_{i+1}}{h_i} \quad (5)$$

S_i can define as $\mathbf{S}_i = \{s_0, s_1, s_2, \dots, s_{n-1}\}$, where $i = 0 \dots n$.

In terms of matrix, it formulates as follows:

$$\begin{bmatrix} h_0 & 2(h_0 + h_1) & h_1 \\ h_1 & 2(h_1 + h_2) & h_2 \\ h_2 & 2(h_2 + h_3) & h_3 \\ & \dots & \\ & \dots & \\ h_{n-2} & 2(h_{n-2} + h_{n-1}) & h_{n-1} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ \cdot \\ \cdot \\ \cdot \\ s_{n-1} \\ s_n \end{bmatrix}$$

$$\begin{bmatrix} f[x_1, x_2] & - & f[x_0 - x_1] \\ f[x_2, x_3] & - & f[x_1 - x_2] \\ f[x_3, x_4] & - & f[x_0 - x_1] \\ f[x_{n-1}, x_n] & - & f[x_{n-2}, x_{n-1}] \end{bmatrix} \quad (6)$$

The previous system is optimal for determining the interpolation point of an open curve according to Cartesian coordinates. The problem is that, when the interpolation process is treated with the opposite previous state, especially with a closed curve, where the interpolation function $y(x)$ is multivalued (Smith, 1992), as illustrated in Fig. 2. In our study, therefore, for solving this problem the interpolation Equation (1) has been reformulated according to the parametric form for each x_i and y_i coordinate as below (Smith, 1992).

$$\beta = Y_i(x) = a_i + b_i(x-\xi) + c_i(x-\xi)^2 + d_i(x-\xi)^3 \quad (7)$$

where $0 \leq \xi \leq 1$

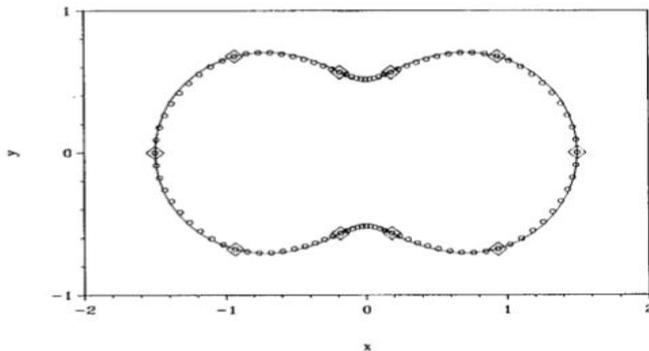


Fig. 2 Multivalued problem with closed curve

Crout reduction method (Rao, 2017) is harnessed for solving the previous nonlinear Equations system (6) problem to estimate the S_i slopes values, as essential values for computing the coefficient fitting $\{a_i, b_i, c_i\}$, in each spine.

Elliptic Fourier Descriptor

Elliptic Fourier Descriptor (EFD) method (Godefroy, 2012) is a simple and robust direct procedure used to obtain Fourier coefficient of harmonic elliptic as invariant features (rotation, scaling and translation) which depends on chain code of contour (Pavlidis, 2012). These features provide a better descriptor depending on applications, especially when they represent boundary of closed shape's contour (Dalitz, et al. 2013). This method is based on Freeman chain code of boundary shape for extracting the Fourier coefficient, where a harmonic coefficient is defined as follows (Dalitz, et al. 2013) (Ballaro, et al. 2002).

$$a_n = \frac{T}{2n^2 \pi^2} \sum_{i=1}^k \frac{dx_i}{dt_i} \left[\cos \frac{2n\pi t_i}{T} - \cos \frac{2n\pi t_{i-1}}{T} \right] \quad (8)$$

$$b_n = \frac{T}{2n^2 \pi^2} \sum_{i=1}^k \frac{dx_i}{dt_i} \left[\sin \frac{2n\pi t_i}{T} - \sin \frac{2n\pi t_{i-1}}{T} \right] \quad (9)$$

$$c_n = \frac{T}{2n^2 \pi^2} \sum_{i=1}^k \frac{dy_i}{dt_i} \left[\cos \frac{2n\pi t_i}{T} - \cos \frac{2n\pi t_{i-1}}{T} \right] \quad (10)$$

$$d_n = \frac{T}{2n^2 \pi^2} \sum_{i=1}^k \frac{dy_i}{dt_i} \left[\sin \frac{2n\pi t_i}{T} - \sin \frac{2n\pi t_{i-1}}{T} \right] \quad (11)$$

Where T refers to the length of chain code.

$$dt_i = 1 + \left(\frac{\sqrt{2}-1}{2} \right) (1 - (-1)^{ui}) \quad (12)$$

$$tn = \sum_{i=1}^n dt_i \quad (13)$$

Proposed Algorithm

The proposed model was implemented in three main phases as described in the algorithm (1). First, the dominant points were created using the RDP recursive algorithm depending on the initial tolerance value ϵ . In the second phase, the parametric cubic spline interpolation model was applied for the interpolation procedure. The previous procedures were executed several times to regenerate a new shape in each iteration based on the new tolerance value. The coordinates of the generated shapes were saved in the testing database. Finally, the elliptic Fourier descriptor was harnessed to compute the Fourier coefficient between those shapes and select the optimal one of them according to the maximum value of correlation between the coefficients of the original shape and the testing shape.

The algorithm for the determination the optimum tolerance approximation value is as follows:

Algorithm 1 Optimum boundary shape selection coding schema

Input: Original shape

Output: Best approximated shape

Step1: Set the tolerance value ϵ and initial value $\epsilon=0$; with maximum value M .

Step2: Read the original shape

Step3: Extract the boundary shape.

Step4: While ($\epsilon < M$) Do

Step5: Increase the value of tolerance $\epsilon = \epsilon + \epsilon$, where $0 \leq \epsilon \leq 1$.

Step6: Compute the dominant points using (RDP) algorithm

Step7: If ($\epsilon > M$) then exit, Else return to Step 5.

Step8: Apply curve fitting interpolation using cubic spline method in parametric form to generate new coordinates.

Step9: Save the new generated coordinates in the coordinate shapes (CS) database.

Step10: End while.

Step11: Use EFD method to calculate the coefficient for each shape on the CS database, and save the results in the coefficient database.

Step12: Apply EFD method for the boundary shape of the original shape to find the coefficient of the original shape.

Step13: Select the optimal tolerance value that provides the maximum value of correlation between the coefficients of the original shape and testing shapes.

Step14: End.

Experimental Results and Discussion

The proposed strategy was evaluated using MPEG-7 database (MPEG-7 Data Set, 2020). The MPEG-7 database is commonly used in polygon approximation analyzing because it contains different patterns of shapes. Trials of the proposed algorithm advanced here were applied to the all MPEG-7 database shapes and complex shapes that have many curves were selected to demonstrate the experimental results.

Fig. 3 (a-c) depicts samples of the original shapes (camel-12, tree-14) readings from the MPEG-7 database. Fig.3 (b-d) illustrates that each boundary shape has been correctly generated by elliptic curve method.

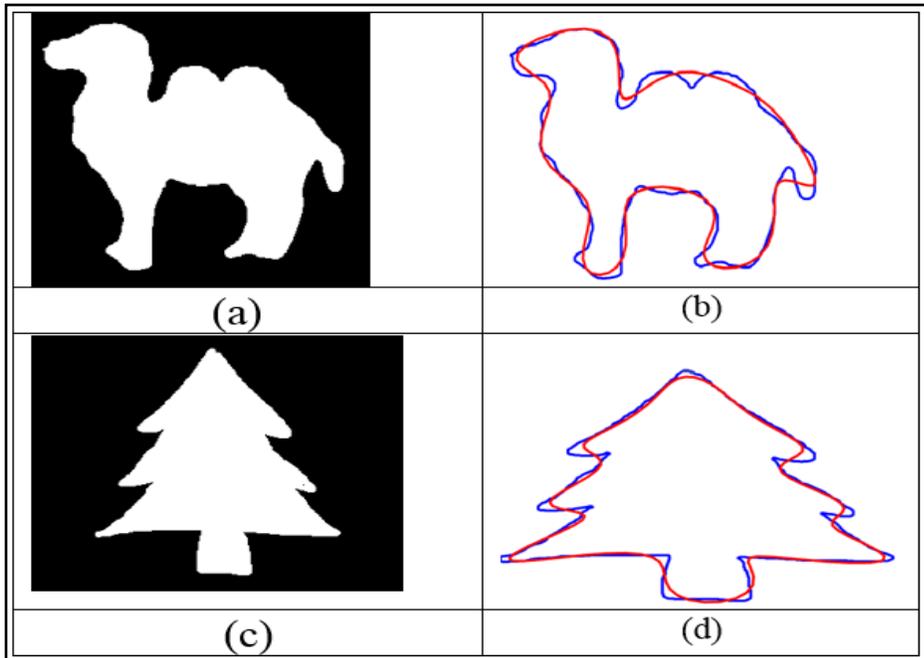


Fig. 3 (a,c) original shapes (camel-12, tree-14), (b,d) boundary shapes generated based on the proposed algorithm

Fig. 4 and Fig. 5 represent the performance of the proposed algorithm for each sample. Shapes from (a to j) were reconstructed based on tolerance intervals in the range of between [0.1-1] and shapes from k to t were reproduced by parametric cubic spline model. For each group, shapes were submitted to (EFD) model for extracting the Fourier coefficient that saved in the coefficient database. Hence, each shape was compared with Fourier coefficient of the original shape, as demonstrated in Fig. 4.

(a) Np=119 ξ=0.1	(b) Np=68 ξ=0.2	(c) Np=54 ξ=0.3	(d) Np=44 ξ=0.4	(e) Np=38 ξ=0.5	(f) Np=32 ξ=0.6	(g) Np=27 ξ=0.7	(h) Np=27 ξ=0.8	(i) Np=25 ξ=0.9	(j) Np=25 ξ=1
(k) ρ=0.9934	(l) ρ=0.9950	(m) ρ=0.9971	(n) ρ=0.9895	(o) ρ=0.9885	(p) ρ=0.9945	(q) ρ=0.9867	(r) ρ=0.9867	(s) ρ=0.9829	(t) ρ=0.9829

Fig. 4. Experimental results for (Camel-12) shape

Note: Np represents number of dominant points

From Fig. 4 and Fig. 5, it can be observed that the largest linear correlation metric (ρ) reflects the optimum tolerance value. The approximated numbers of dominant points are (54, 21), which were depended on the largest correlation (ρ) values which are (0.9971, 0.9850). Overall, the optimum tolerance values for shapes were between 0.3 and 0.4.

(a) Np= 71 $\xi=0.1$	(b) Np=32 $\xi=0.2$	(c) Np= 24 $\xi=0.3$	(d) Np=21 $\xi=0.4$	(e) Np= 18 $\xi=0.5$	(f) Np=18 $\xi=0.6$	(g) Np=16 $\xi=0.7$	(h) Np=14 $\xi=0.8$	(i) Np=13 $\xi=0.9$	(j) Np=13 $\xi=1$
(k)	(l)	(m)	(n)	(o)	(p)	(q)	(r)	(s)	(t)
$\rho=0.9837$	$\rho=0.9785$	$\rho=0.9782$	$\rho=0.9850$	$\rho=0.9686$	$\rho=0.9686$	$\rho=0.9667$	$\rho=0.9845$	$\rho=0.9816$	$\rho=0.9816$

Fig. 5 Experimental results for (Tree-14) shape

The current model can reproduce the shapes with good quality based on a very limited number of dominant points such as 13 points, as reported in Fig. 5. Also, the tolerance value could be increased up to 1 and still achieve a good shape quality, as reported in Fig. 5 and Fig. 4.

One of the reasons behind the successful performance of the current model is its capability to generate various optimum tolerance values given by the elliptic curve algorithm to improve the generality of the final value.

The recommendation advanced in this investigation is that the proposed model is particularly well suited for the shapes detection or reduction based on minimal data captured from any visual segment.

Conclusions

Previous studies generated shapes based on determining the optimum dominant points. In contrast, the current research advances a model for reconstructing shapes based on determining the optimum tolerance value in a simple and robust way. This paper presents a new approach based on the Min -# Polygon approximation measurement system. Using features that can be extracted from the original shape and fed into a simple direct descriptor algorithm, improved the detection of optimum tolerance value. The results indicate that the suggested method determines the optimum tolerance value using minimal dominant points

that reconstruct the original shape with a high level of accuracy. Overall shapes readings from the MPEG-7 database the optimum tolerance values are between 0.3 and 0.4.

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