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RECTANGULAR WAVEGUIDE DISCONTINUITY ANALYSIS USING FEM

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ABSTRACT
For computational analysis of a wide range of electromagnetic problems the Finite Element Method (FEM) is a renowned tool. FEM basically is a technique used to obtain approximate solution of boundary value problems for different complex structures. In this paper FEM is implemented and used to analyze the Eigen Modes of 2D waveguide structures, the accurate results presented are in excellent agreement to analytical ones wherever applicable. Also the effect of discontinuity due to airgap between two waveguide in waveguide is analyzed in terms of variation in field patterns and transmission coefficient.

Keywords: Eigen Modes and Eigen Values, Waveguide, Finite Element Method, Time-Domain Method

INTRODUCTION
The finite element method (FEM) is very well known as one of accurate and appropriate methods for the computational analysis of complex field problems with boundary conditions. The method easily copes with complex geometries and approximations of higher order solutions. The large success of FEM is based on the simple finite element procedures used, which consist of formulating the problem in a different form, finite element discretization of obtained form and solving the resulting finite element equations. Basically, it is a mathematical based technique to find solutions to partial differential equation (PDE) and integral equations. For solving different partial differential equations, the first goal is to find an equation that come close to the equation under study and that should be numerically stable. FEM is a very good technique for solving PDE in a complex domain where the domain changes as the values change throughout the domain. The approach of solution is based on either eliminating the differential equation entirely or modifying the partial differential equation to an equivalent ordinary differential equation, this equation can be solved using other

regular methods such as finite differences, etc. So to analyse different electromagnetic problems it is a very good tool.

Now the main focus is on how the finite element works so to use FEM the specified area is divided into small structures known as elements by this technique the behaviour of each element can be easily and clearly well-defined through few PDE. The obtained differential equations are changed to algebraic equations and these algebraic equations are then converted to matrix equations which are appropriate for solution on computer. These elements are combined to obtain a structural equation. With the same approach of solution is based on either eliminating the differential equation entirely or modifying the PDE to an equal ordinary differential equation the matrix is being resolved and the deflection of all nodes were calculated. Several other elements and a node can be shared or we can say a node can be shared by numerous other nodes and the deflection at a shared node signifies the deflection of the element which is being shared at the node position. For more precise study in FEA, choosing the right elements is more important.

In this work, a technique for solving waveguide problems is proposed. A mathematically efficient FEM formulation is used that shows propagation modes and can be used to analyze problems which are involving linear systems of random complex permittivity and permeability. Solving these eigenvalue problems results in approximate fields for all components of the various eigenmodes in the waveguide, these eigenmodes can be further used to obtain the corresponding eigenvalues [1,2]. The paper presented a comparison of the proposed approach with available results to clarify the reliability and accuracy of the method. This work also analyses the effects of the air gaps in between the junction of two waveguides this effect of discontinuity due to airgap between two waveguides is analysed in terms of variation in field patterns and transmission coefficient.

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FEM ANALYSIS OF WAVEGUIDE

The FEM is widely used in the examination of waveguide components. The propagation characteristics of all different shaped waveguides of various compositions are easily achievable by this method. The FEM is based on spatial discretization [1]. By using this approximation, it is possible to manipulate waveguide cross-sectional geometries that are very much similar to real structures used in any practical device. These complex structures are not suitable for analytical solutions. Consequently, FEM represents a favorable tool to distinguish and solve such problems [2, 4]. Modern phased array radars require polarization flexibility requirements for broadband array elements like a rectangular waveguide.

For this work, an element is assumed having a triangular shape as shown in Fig.1 and the whole cross section of the rectangular waveguide is divided into a number of finite elements [3].

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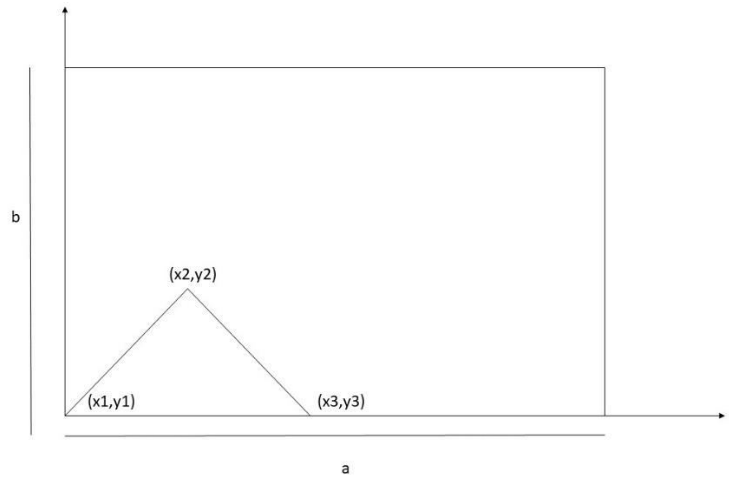

Fig.1 Finite element formulation

Assume a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3). $\vec{u}$ and $\vec{v}$ are two vectors which are joining the vertices $[(x_1, y_1), (x_2, y_2)]$ and $[(x_1, y_1)$ and $(x_3, y_3)]$. The two integrals obtained when we uses the FEM are:

First

$I_1 = \int_\Delta V^2_{(x,y)} dx.dy$

Second

$I_2 = \int_\Delta |\nabla V|^2 dx.dy$

Now

$I_1 = \int_\Delta V^2_{(x,y)} dx.dy = \int_\Delta (ax + by + c)^2 dx.dy$

Now replacing the value of (x, y),

$I(\phi) = \frac{sin\alpha}{2} \int_0^{d_1} \int_0^{d_2} \left[a\left\{x_1 + \frac{u(x_2-x_1)}{d_1} + \frac{v(x_3-x_1)}{d_2}\right\}\right] du.dv$

and finally,

$$\int_0^{d_1} \int_0^{d_2} du.dv = d_1.d_2$$

We also have

$$\nabla V(x, y) = (a, b)$$

and hence

$I_2 = \int_\Delta |\nabla V|^2 dx.dy = \frac{(a^2+b^2)d_1.d_2 sin\alpha}{2}$

These two integrals ($I1$ and $I2$) for each element are calculated. The summation of these integrals will be calculated for all the elements in which cross-section have been divided.

A. Calculation of Eigenvalues

The eigenvalues of the matrix are as follows

$$(\mathrm{V^T AV} - \mathrm{k^2 V^T BV})$$

When minimized over V (vector of vertex nodal value) gives the quadratic form defined by

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$\int_{\Delta} |\nabla V|^2 dx.dy - k^2 \int_{\Delta} V^2 dx.dy$

Here, $\delta \int_{\Delta} (\nabla \vec{V}, \nabla \vec{V}) dx.dy = 2 \int_{\Delta} (\vec{\nabla}, \delta \nabla \vec{V}) dx.dy$

$$= -2 \int_{\Delta} \delta V. \nabla^2 V \, dxdy$$

and

$$\delta \int_{\Delta} V^2 dx.dy = \int 2V.\delta V dxdy$$

From equation, $-2 \int \delta V. \nabla^2 V \, dxdy - 2k^2 \int \delta V.V \, dxdy = 0$

$$\int \delta V (\nabla^2 + \mathrm{k}^2) V dxdy = 0$$

$$(\nabla^2 + \mathrm{k}^2) V dxdy = 0$$

A calculation of equation using the FEM gives

$$AV - k^2 BV = 0$$

$$\Rightarrow \quad (A - k^2 B)V = 0$$

$$\Rightarrow \quad |A - k^2 B| = 0$$

The solution of above matrix will provide the eigen values.

B. Practical Approach

The practical approach of calculation is as follows:

1. The whole cross section is divided into finite elements generally triangular in shape.

2. Coefficients of finite elements can be calculated using substitute surface of nth finite element A(n) and angles at vertexes of nth element to these relations coefficients for finite elements can be obtained, after that diagonal matrices for isolated finite elements can be obtained.

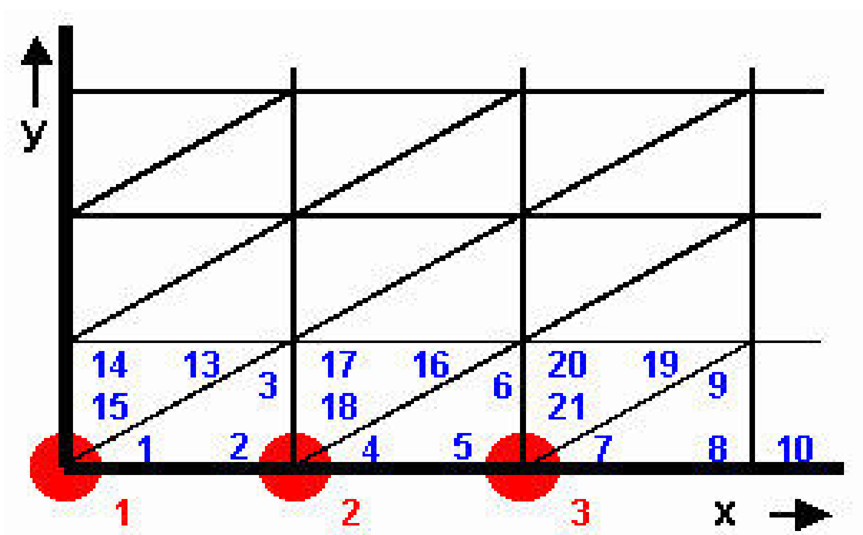

Fig.2 Mesh grid

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3. A matrix is formed, which defines mutual relations among local nodes shown as blue number is Fig 2 and global ones shown as red numbers in Fig 2. Obtained matrix columns are global nodes and rows are local nodes.
   With mathematical concept it is described as
$$S_C = C^T SC, \quad T_C = C^T TC$$
4. Solving the final matrix equation
$$\mathrm{SE} + \mathrm{k}^2\mathrm{TE} = 0$$

Critical wave numbers of modes k and longitudinal component of electric-field intensity vector E of those modes can be obtained.

TIME-DOMAIN METHOD: FDTD

For problems solving approach in electromagnetics the finite difference method in the time domain (FDTD) is probably the simplest technique both theoretically and in terms of implementation. It can be used to solve a wide range of different problems. Although FDTD method can be used to solve various complex problems, but it is generally computationally expensive. In FDTD the solution approach can require a large amount of system memory and computing time. The FDTD technique fits roughly into the group of "resonance region" techniques, i.e., those techniques in which the characteristic dimensions of the area of interest are nearly in the order of a wavelength. If the object is very small in comparison to the wavelength, quasi-static approximation mostly provides a more efficient solution.

However, if wavelength is very small as compared to the physical properties of waveguide, beam-based methods or some other techniques may deliver a further efficient way of solving the problem.

A. Finite Element and Finite Difference Method Comparison

The major dissimilarities between FEM and FDM are as follows:

1. The main feature of the FEM technique is its capability to handle complex bodies (boundaries) relatively easily. While FDM is limited to rectangular shapes and their simple transformations.
2. The main property of finite differences is that they are very easy to use.
3. In many ways FDM can be considered a subset of the FEM. In both methods, the estimates are defined over the entire area, but it is not need to be continuous. Alternatively, the function can be defined on discrete domains, but this approach is not FEM.
4. There are various reasons to deliberate the mathematical basis of the FEM approximation more reasonable, because the class of the approximation between the grid points is not good in FDM.
5. On the other hand the quality of the FEM approximation is generally higher than in the corresponding FDM approach, but it is totally problem dependent.

SIMULATION RESULTS

After running the simulation code on MATLAB following are the results:
Meshgrid of the square waveguide is seen in Fig 3. Lowest cutoff wave number k_c of TM_{11} mode in square waveguide is calculated 4.4976.

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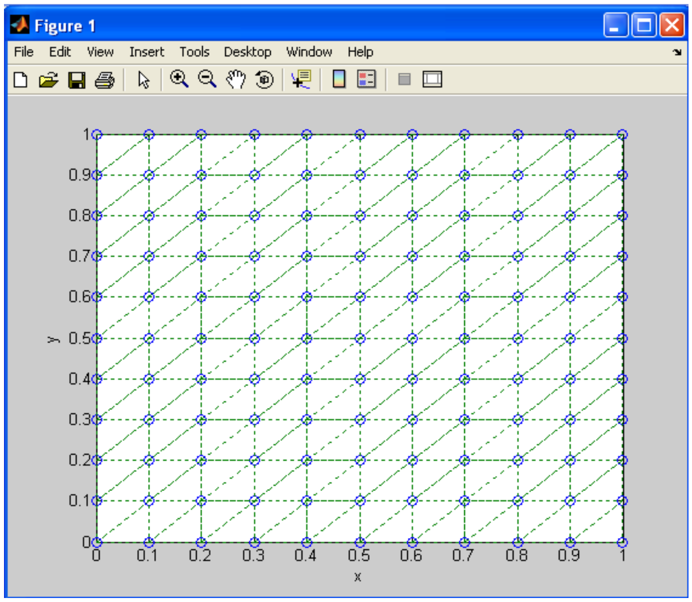

Fig.3. Meshgrid of square waveguide

Nx	***Ne***	***Kcalc***	***% error***
10	***200***	***4.4976***	***1.2316***

Fig. 4 shows the plot of various Eigen modes of waveguide for length L. In Fig 5 we can see the comparison in finite element solution and analytical solution of wave equation.

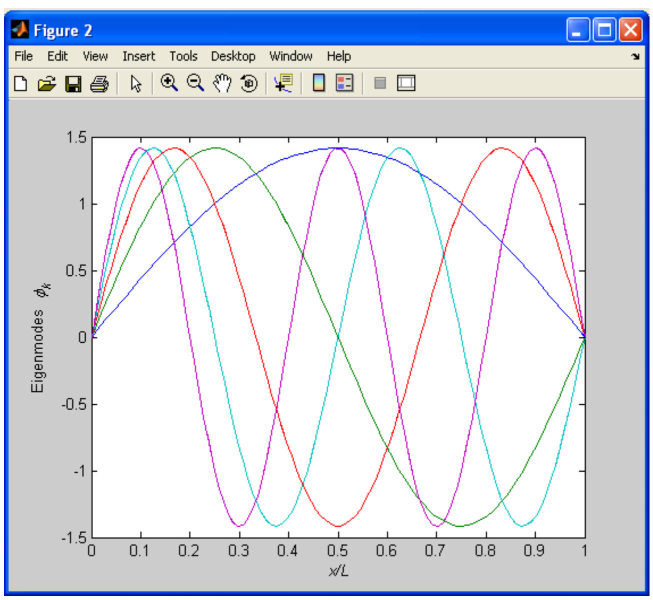

Fig.4 Eigenmodes values

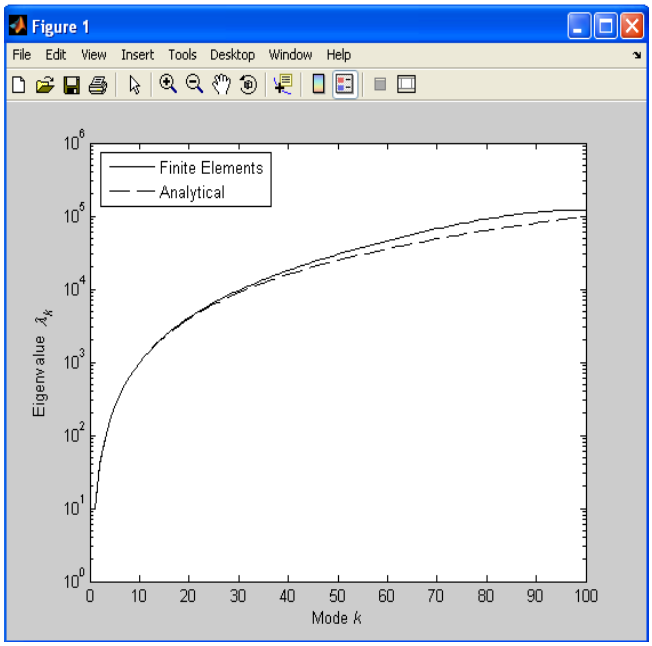

Fig.5 Comparison of Finite element and analytical solution
In Fig 6 solution of wave equation using FEM is given.

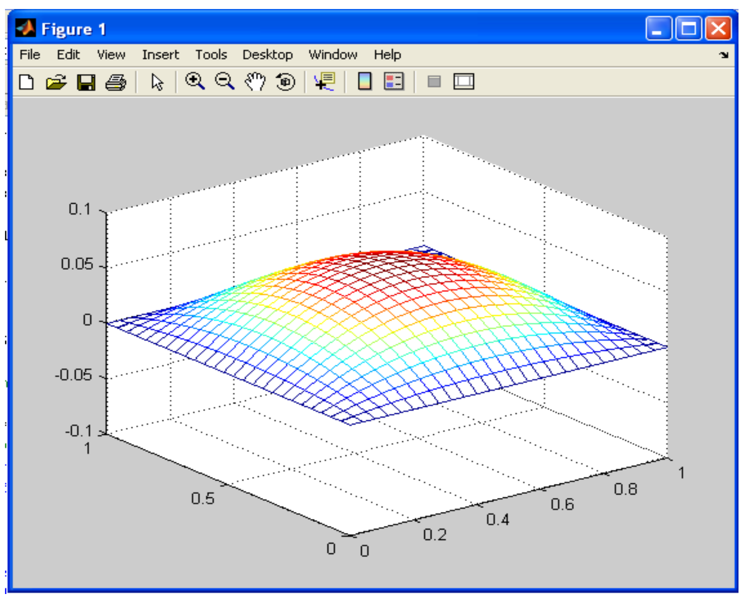

Fig.6 Plot solution of wave equation

After that analysis of waveguide Discontinuity will be studied and its mesh plot is given in Fig.7. Its field intensity real part of Ez and real part of Hx and Hy is given in Fig 8 and Fig 9 respectively.

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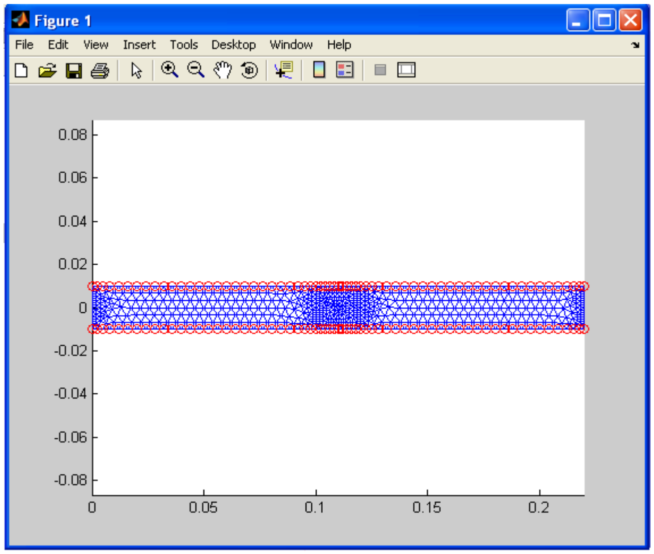

Fig.7. Mesh plot

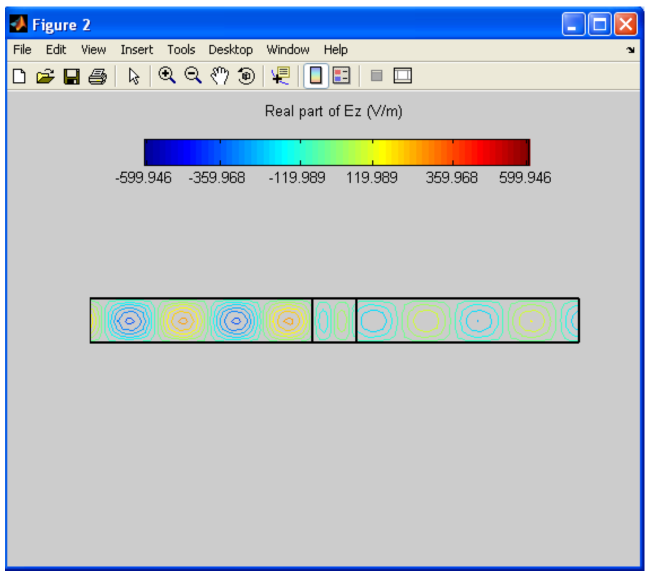

Fig.8 Field intensity real part of Ez

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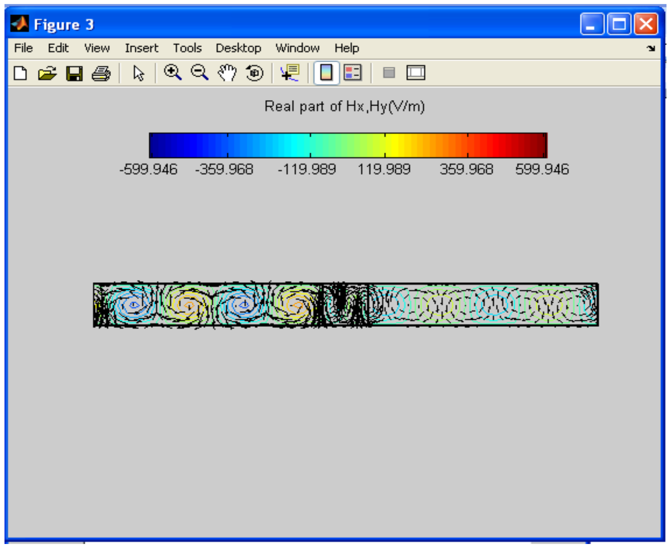

Fig.9 Field intensity real part of Hx and Hy

With the help of calculated electric and magnetic field patterns an attenuation of approx10db is measured due to discontinuity in the junction of two rectangular waveguides.

CONCLUSION

A FEM technique to analyze the Eigen modes of 2D waveguide structures has been presented in the paper. The obtained results are in excellent agreement with earlier published data. Also the procedure is described to find reflection and transmission coefficient of rectangular waveguide discontinuity. Approx. 10db attenuation is measured due to discontinuity in junction of the waveguide. The procedure described can be used to handle the air gap discontinuity present between the junctions of any two rectangular waveguides.

REFERENCES

1. Silvester, P. P., & Ferrari, R. L. (1996). *Finite elements for electrical engineers*. Cambridge university press.
2. Vaish, A. and Parthasarathy H,(2006) "Numerical computation of the modes of electromagnetic wave propagation in anon-homogeneous rectangular waveguide," *IEEE International Conference*, INDICON-06,
3. Sadiku, M. N. (2000). Numerical techniques in electromagnetics. CRC press..
4. Singh, V., Prasad, B., & Ojha, S. P. (2001). Modal analysis and dispersion curves of an annular waveguide having a guiding-region cross section bounded by two lemniscates of Bernoulli. Microwave and Optical Technology Letters, 28(1), 75-78.
5. Wexler, A. (1967). Solution of waveguide discontinuities by modal analysis. IEEE Transactions on Microwave Theory and Techniques, 15(9), 508-517.
6. Itoh, T. (Ed.). (1989). Numerical techniques for microwave and millimeter-wave passive structures. Wiley-Interscience.
7. Pozar, D. M. (2011). Microwave engineering. John wiley & sons.